Mills’s The Grand Unified Theory of Classical Physics (GUTCP) shows amazing equations and calculations that indicate his theory is correct.
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Niels Bohr was close. In 1913, using classical physics, he proposed a model of the hydrogen atom that produced equations that matched the spectrographic light emissions from hydrogen atoms. But the model failed when applied to other experimental data.

So Standard Quantum Physics (SQM) was invented but it uses complex equations to describe the atom. Few people really understand Standard Quantum Mechanics, including most physicists. It can be considered curve fitting since there is no unifying methodology in the derivation of the equations.

Randell Mills has come up with a theory named The Grand Unified Theory of Classical Physics (GUTCP) which uses classical physics to describe the atom that produce equations that match all the experimental data far better than the previous two models listed. Equations produced using GUTCP are based on classical physics and special relativity that are applied in a consistent way and result in much simpler equations than Standard Quantum Mechanics.

For example, in the Bohr Model and SQM, the hydrogen is at its smallest size at principal quantum number \( n = 1 \). But in Mills’s GUTCP, the electron in the hydrogen atom can release energy as it drops to smaller sized fractional orbit states (hydrinos) such as \( n = 1/2 \) or \( n = 1/3 \) or \( n = 1/4 \) etc. The smallest theoretical orbit state is \( n = \alpha = 1/137.035999 \), (i.e. the fine structure constant) where the electron in the hydrogen atom would orbit at the speed of light \( c \) if it could reach that orbit state.

During particle production where a photon is converted into an electron and a positron, the electron and the positron initially exist as a transition state orbsphere (TSO) that both immediately ionize to a free electron and a free positron. The transition state orbsphere (TSO) has multiple energy equations associated with it that use the fine structure constant, \( \alpha \), in the calculation. Specifically there are five different energy equations for the TSO that match Einstein’s energy equation for the electron:

\[
E = mc^2 = 510998.896 \text{ eV}.
\]
Those five energies are:

1. Planck equation energy = 510998.896 eV
2. Resonant energy = 510998.896 eV
3. Electric potential energy = 510998.896 eV
4. Magnetic energy = 510998.896 eV
5. Mass/Spacetime metric energy = 510998.896 eV

These five energies occur at different times during the process that a photon is converted into an electron and therefore the law of conservation of energy is preserved.

The most dramatic method of showing this is by graphing the different energies of the electron in the hydrogen atom as a function of orbit state \( n \). Figure 1 on the next page shows that the energies all converge to the rest mass of the electron (510998.896 eV) at orbit state \( n = \alpha = 1/137.035999 \) (i.e. the orbit state of the TSO). The equations that are graphed are shown on page 14 of this document which is a single page taken from a PowerPoint presentation from Blacklight Power.

Figure 3 shows that when the same equations are adjusted to correspond to the Bohr Model (i.e. using the postulates of the Bohr Model), there is no intersection at 510998.896 eV except for the Electric Potential Energy (note that the other four energies intersect at the square root of the fine structure constant which is 1/11.7). The right hand axis shows the velocity of the electron which reaches the speed of light \( c \) at \( n = \alpha = 1/137.035999 \).
The particle production energies listed at the top of the previous page are graphed above in Figure (1): the Spacetime Metric Energy, Planck Equation Energy, Resonant Energy, Electric Potential Energy and the Magnetic Energy. The equations are graphed by replacing $\alpha$, (alpha) the fine structure constant, in the particle production energy equations from page 14 of this document with orbit state $n$ and the result is all of the equations intersect at the rest mass of the electron, 510998.896 eV, at $n = \alpha$ where the radius is: $r = \alpha a_0$. The energies do not exist at the same instant of time during particle production since that would violate the law of conservation of energy.

The particle production equations on page 14 are non-relativistic in the sense that the Lorentz factor is not directly used in the derivation or the final equations. Alternatively, the relativistic energy equations for the electron in the hydrogen atom are described in Appendix A, B and C of this document.
Potential Energy (PE), Kinetic Energy (KE) and Binding Energy ($E_B$) graphed in Figure (2) are from equations on page 46 of this document.
The particle production energies listed on page 5 (and page 14) are graphed in Figure (1): the Spacetime Metric Energy, Planck Equation Energy, Resonant Energy, Electric Potential Energy and the Magnetic Energy. These particle production equations are non-relativistic in the sense that the Lorentz factor (listed on next page) is not directly used in the derivation of the final equations. Alternatively, see Appendices (A), (B) and (C) for the relativistic equations. Figure (1) is created by replacing alpha (α), the fine structure constant, in the particle production energy equations from page 14 with orbit state \( n \) and the result is all of the equations intersect at the rest mass of the electron, 510998.896 eV, at \( n = \alpha \) where the radius is: \( r = \alpha a_0 \). The equations were derived specifically for electron particle production and the fact that the energies don’t equal each other at other orbit states and radii indicates that the particle production radius, \( r = \alpha a_0 \), is the only radius where an electron/positron pair can be created from a photon. The top of the vertical gridlines in Figure (1) are labeled with the radius and the bottom is labeled with orbit state \( n \). The energies do not exist at the same instant of time during particle production since that would violate the law of conservation of energy (also, the spacetime metric energy is not an energy).

GUTCP states that the free electron (unbound to a nucleus) can be modeled as an infinite number of infinitesimal sized masses and charges having the shape of a very thin flat circular disk as shown below left. And the electron orbitsphere is a free electron formed into a spherical shape with the infinitesimal charges and masses contained in a thin shell of orbiting mass and charge as shown below right.
How could the energies in Figure 1 be interpreted?
The energies graphed in Figure (1) are based on equations that do not include the Lorentz factor directly in their derivation. Either one or some of the energies in Figure (1) might (with an emphasis on might since the following is not in GUTCP) represent the energy of the free electron if it were adiabatically compressed from \( n = \infty \) (i.e. a free electron) down to its particle production radius at \( n = \alpha \) where the radius is \( r = \alpha a_0 \).

Figuratively speaking this could be accomplished using an infinite number of movable infinitesimal sized mirrors that compress the free electron from its flat disk shape into a spherical shape and then down to its particle production radius. A graph of its orbiting kinetic energy versus radius might look similar to the magnetic energy graphed in Figure (1) since magnetic energy and kinetic energy are similar concepts. At \( r = \alpha a_0 \) (the particle production radius) the infinitesimal masses and charges orbit at the speed of light \( c \).

Using infinitesimal mirrors to compress the electron (in this hypothetical scenario) would require 510998.896 eV of energy put into the mirrors for the compression. If the mirrors were instantly removed then the electron would expand back to its original flat disk shape and exist as a free electron and 510998.896 eV worth of compression energy would be released in some manner (such as thermal energy) since energy from the compression has to be counted. Alternatively, if the compressed electron were slowly expanded by moving the mirrors outward slowly and if the energy of the restraining force (i.e. from Energy = Force x Distance) were able to be captured in a battery, then the battery would gain 510998.896 eV.

How could the relativistic equations for the electron energy shown in Appendices (A), (B) and (C) be interpreted?
The relativistic equations for the electron in the hydrogen atom incorporate the Lorentz factor equation directly in their derivation:

\[
\gamma = \frac{1}{\sqrt{1 - \left( \frac{v}{c} \right)^2}}
\]

The relativistic equations in Appendix (A), (B) and (C) represent the energy of the electron as it falls towards the proton due to electrostatic attraction and emits photons in the process. Graphs of these energies are shown in Figures (A1)-(A5) in Appendix (A) and Figures (B1) and (B2) in Appendix (B).
When equations from page 14 are graphed using the Bohr Model assumptions with fractional orbit states there are *no* intersections at 510998.896 eV (except for Electric Potential Energy).

Figure 3. Bohr Model Energies at Fractional Orbit States (though fractional orbit states are not allowed in Bohr model)

510998.896 eV = rest mass of electron = $mc^2$
For decades, physicists have struggled with how to interpret the fine structure constant:

Fine Structure Constant \( (\alpha) = \alpha = \frac{1}{137.035999} \)

In 1985, Physicist Richard Feynman said the following:

“\textit{It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.}”

”\textit{It’s one of the greatest damn mysteries of physics: A magic number with no understanding by man}”

\[
\text{Fine structure constant } = \alpha = \frac{e^2}{\hbar c (4\pi\varepsilon_0)} = \frac{e^2 c \mu_0}{2\hbar} = \frac{1}{137.035999}
\]

Where:
- \( c \) = speed of light
- \( e \) = elementary charge
- \( h \) = Planck constant
- \( \hbar \) = reduced Planck’s constant
- \( \varepsilon_0 \) = permittivity of free space
- \( \mu_0 \) = the permeability of free space
fine structure constant \( \alpha = \frac{1}{137.035999} \)

The value of the fine structure constant and where it comes from is explained by Randell Mills’s GUTCP model of the atom. In GUTCP, the principal quantum number \( n \) for the hydrogen atom can take on integer and fractional values:

\[
\text{allowed orbit states for} \quad n = \left\{ \begin{array}{c}
1,2,3,4 \ldots \text{infinity} \\
\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{p}
\end{array} \right. \quad \text{and} \quad p \leq 137
\]

Mills’s GUTCP terms the hydrogen atom an “electron orbitsphere” which is a stable hydrogen atom with an electron orbiting a proton at the center and has a spherical shape. It can have a stable fractional orbit state as low as 1/137 according to the “allowed orbit states” listed above. But the absolute lowest orbit state is reserved for a special case of the electron orbitsphere (termed the transition state orbitsphere or TSO) which occurs during pair production and is orbit state (i.e. principal quantum number) \( n = \alpha = 1/137.035999 \). The transition state orbitsphere at \( n = \alpha \) has the following properties:

- There is no \textit{proton} at the center but there is a \textit{positron} (the anti-electron) that supplies the central electric field (but see note at bottom of page 16 regarding this central attractive force).
- The electron orbits the positron at the speed of light \( c \) and the orbit velocity is 137.035999 times faster than normal hydrogen in the ground state at \( n = 1 \).
- The radius of the TSO is 137.035999 times smaller than normal hydrogen in the ground state.
- The electron orbit frequency matches the frequency of a photon having an energy of 510998.896 eV (i.e. the rest mass of the electron).
This document focuses mainly on page 18 from a Blacklight Power document that can be found on the website www.blacklightpower.com. The link is:


Page 18 of that pdf is shown on the next page.

Also, more details can be found in Mills’s GUTCP book, The Grand Unified Theory of Classical Physics (GUTCP). Search for “Pair Production” (Chapter 29). Additional details are also in “Gravity” (Chapter 32) which has a sub section “Particle Production”.
Relationship of the Equivalent Particle Production Energies

When the orbitsphere velocity is the speed of light:

Continuity conditions based on the constant maximum speed of light given by Maxwell’s equations:

(Mass energy = Planck equation energy = electric potential energy = magnetic energy = mass/spacetime metric energy)

\[ m_0 c^2 = \frac{\hbar \omega^*}{m_0 \kappa_C^2} = \frac{\hbar^2}{m_0 \kappa_C^2} = \alpha^{-1} \frac{e^2}{4 \pi \varepsilon_0 \kappa_C} = \alpha^{-1} \frac{\pi \mu_0 e^2 \hbar^2}{(2 \pi m_0)^2 \kappa_C^3} = \frac{\alpha h}{1 \text{sec}} \sqrt{\kappa_C c^2} = E_{\text{mag}} = E_{\text{spacetime}} \]


rest mass energy of electron

This is an amazing result of Randell Mills’s classical physics!


Table 1. Particle production energies and the transition state orbitsphere.

<table>
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<tr>
<th>#</th>
<th>Energy</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Planck equation energy.</td>
<td>510998.896 eV is the minimum energy of a photon required to create the electron in the TSO and has a wavelength that matches the circumference of the TSO.</td>
<td>A superimposed photon of at least 510998.896 eV creates the positron so that the minimum total energy of the photon required is 1.02 MeV (i.e. twice 511 keV)</td>
</tr>
<tr>
<td>2</td>
<td>Resonant energy (photon energy equivalent).</td>
<td>A volume of free space equal to ½ the volume of the TSO has a resonant energy equal to 510998.896 eV.</td>
<td>The other ½ of the TSO volume resonates with 510998.896 eV and creates the positron. Total resonant energy equals 1.02 MeV.</td>
</tr>
<tr>
<td>3</td>
<td>Electric potential energy.</td>
<td>Electric potential energy of the electron in the hydrogen atom evaluated between TSO radius and infinity is 510998.896 eV.</td>
<td>There is no proton at the center of the TSO but there is a positron.</td>
</tr>
<tr>
<td>4</td>
<td>Magnetic energy.</td>
<td>Energy in magnetic field of TSO from orbiting negative charge currents is 510998.896 eV.</td>
<td>Same applies to positron where energy in magnetic field of TSO from orbiting positive charge currents is 510998.896 eV.</td>
</tr>
<tr>
<td>5</td>
<td>Mass/Spacetime metric energy.</td>
<td>Mass/Spacetime metric energy ties together electric, magnetic and gravitational energy.</td>
<td></td>
</tr>
</tbody>
</table>
Notes regarding Table 1:

- The energies in Table 1 are based on the equations from page 14 of this document.
- The rest mass of the electron is 510998.896 eV according to the mass energy equation:
  \[ E = m_0 c^2 = 510998.896 \, \text{eV} \]
- The energies listed in Table 1 do not occur at the same instant of time because that would violate the law of conservation of energy. Each energy occurs in a step by step fashion and at a different point in time.
- The resonant energy in Table 1: “2. Resonant energy (photon energy equivalent)” is an energy calculated using the photon energy equation (Planck equation) applied to the resonant frequency of ½ the volume of the TSO.
- GUTCP does not mention that ½ of the TSO volume resonates with the electron and the other ½ resonates with the positron. I only base that on the fact that the resonant frequency equation uses ½ of the capacitance and ½ of the inductance of a sphere that is the size of the TSO. Details are shown in the derivation of that equation in this document.
- The “3. Electric potential energy” uses a formula that matches the hydrogen atom with the proton at the center while the TSO does not have a proton at the center. But at one instant of its life, the TSO is made of two concentric spherical orbitspheres – a positively charged positron and a negatively charged electron. The positively charged positron takes the place of the proton in supplying the central* electric field and the electric field is in a small gap between the two orbitsphere shells. The electric field energy between these two shells can be written in a form that matches the form for the hydrogen atom with the proton at the center. GUTCP derives the electric potential energy for the TSO starting at GUTCP Eq. (29.1) and ending with GUTCP Eq. (29.10).

*The central electric field between the positron and the electron at particle production most likely can not be modeled using conventional equations for a static, steady state electric field. The reason is particle production is not a steady state process and the positron and electron form from a photon and immediately ionize to a free positron and a free electron. So describing the positron as supplying the central electric field that replaces the proton in the hydrogen atom is most likely incorrect and oversimplifying the math.
Pair Production: the conversion of energy into matter.

In conventionally accepted physics, a photon can be converted into an electron and a positron (the anti-electron) when it strikes a nucleus and is known as “Pair Production” or “Particle Production”. Quantum Mechanics describes this in a messy and confusing way. But the GUTCP equations that describe this are classical, meaning no quantum theory is involved and it uses Newtonian dynamics and Maxwell’s equations and includes Einstein’s Special Relativity. Much of it can be understood by people with a medium knowledge of physics.

The fact that 5 completely different equations involving the radius of the TSO equate to exactly the rest mass of the electron strongly indicates that Mills has the right theory.
Pair Production

• Photon converted into free electron and free positron (i.e. a pair).
• Requires two superimposed photons each having at least 511 keV that have opposite circular polarization (total 1.022 MeV).
• The superimposed photons strike a nucleus such as a proton or a group of bonded protons and neutrons. The photon has linear momentum which is conserved in the collision with the nucleus.
• “Transition state orbitsphere” (TSO) is created at orbit state $n = \alpha = 1/137.035999$ (i.e. the fine structure constant) with radius equal to: $r = na_0 = \alpha a_0$
• For an electron at $n = \alpha$, “matter and energy are indistinguishable by any physical property” according to GUTCP.
• The transition state orbitsphere lasts for a fraction of a second before creating an electron and a positron that then ionize to infinity. Creates free unbound electron and free unbound positron each having a mass of 510998.896 eV (total mass created equals 1.022 MeV)
• Any excess energy above 1.022 MeV that the photon had is converted into kinetic energy of the nucleus that was struck and the kinetic energy of the positron and electron created (possibly other lower energy photons, but maybe that would violate conservation of angular momentum? I’m not sure).
Conversion of energy into matter (photon converted into an electron).

The steps for the conversion of energy (a photon) into matter (an electron and positron) are the following:

**Step 1.** A photon having an energy equal to or greater than 1.02 MeV strikes some type of nucleus such as a proton. Photon is made of two oppositely circularly polarized photons having at least 511 keV each.

**Step 2.** This collision causes a volume of space equal to the TSO to resonate between electric and magnetic energy. This resonant frequency matches the frequency of a 510998.896 eV photon.

**Step 3.** TSO is created and has two superimposed magnetic field energies of 510998.896 eV (one for the electron and the other for the positron) from the positive and negative electric charge traveling in a circular path on the TSO at the speed of light. The orbit frequency of the charge currents on the TSO matches the frequency of a 510998.896 eV photon.

**Step 4.** The electron velocity (surface charge current velocity) slow down to zero as it ionizes to infinity and the magnetic field energy also drops to zero. The change in electric potential energy is: 2 x 510998.896 eV which equals 1.022 MeV at infinite distance apart.

The overall result is that a 1.022 MeV photon (minimum) has been converted into a free ionized electron and a free ionized positron that both have a rest mass of 510998.896 eV.
Definition of the Electron Orbitsphere (for the hydrogen atom with one electron orbiting one proton):
In GUTCP, the electron orbitsphere is a spherical shaped thin shell of negative electric charge that surrounds the positive proton at the nucleus. Charge currents orbit on an infinite number of circular paths around this sphere and the sum of the charge currents amounts to the charge of an electron, \(-1e\) (or \(-1.6021 \times 10^{19}\) Coulombs).

Trapped photon (not shown) inside infinitely reflective sphere

electron can be modeled as a shell of negative charge made from an infinite number of infinitesimal sized charges and masses orbiting on great circles

radius: \(r = na_0\)

stable (allowed) orbit states:

<table>
<thead>
<tr>
<th>(n)</th>
<th>(1, 2, 3 \ldots ) infinity</th>
<th>(normal hydrogen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>(\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{p}) where (p \leq 137)</td>
<td>(hydrinos)</td>
</tr>
</tbody>
</table>
Definition of the Transition State Orbitsphere (TSO):
The transition state orbitsphere (TSO) is a special case of the electron orbitsphere with principal quantum number (i.e. orbit state) \( n = \alpha = 1/137.035999 \) (alpha, the fine structure constant) and is the smallest allowable principal quantum number for the electron in GUTCP. The TSO has a radius \( r = n a_0 = \alpha a_0 \) where \( a_0 \) equals the Bohr radius which means that the TSO is 137.035999 times smaller than normal hydrogen at principal quantum number \( n = 1 \) (and the electron orbits at a velocity that is 137.035999 times faster than the \( n = 1 \) orbit state). There is no proton at the center of the TSO, instead there is a positron (anti-electron) that provides the central attractive force for the electron (but see note at bottom of page 16 regarding this central attractive force). The positron and the electron are both orbitspheres that are temporarily superimposed on top of one another with one of them having a radius that is slightly smaller than the other. This is a temporary orbit state during pair production that lasts for a fraction of a second before the electron and positron both ionize out to infinity (i.e. separate from each other to an infinite distance).
Two superimposed photons create the TSO with each one having a minimum of 510998.896 eV. The photons are oppositely circularly polarized having a total of 1.022 MeV (minimum) where one creates the electron, the other creates the positron.

The next pages go through the steps of Pair Production and the equations involved.

**Pair Production**

1. Planck Equation Energy \[ \hbar \omega = m_0 c^2 = 510998.896 \text{ eV} \]

Details:
The photon strikes any nucleus (such as a proton or a group of bonded protons and neutrons). The linear momentum of the photon is conserved in the collision with the nucleus.

(2) X 511 keV (minimum) photons that total 1.022 MeV

Rest mass of the electron.

GUTCP Eq. (32.48b)
Proof is seen by inserting the equations below into the Planck energy equation on the previous page and making the following two assumptions
1. The photon frequency matches the orbit frequency of the charge currents orbiting on great circles of the TSO.
2. The charge currents travel at the speed of light \( c \) (see orbitsphere surface current velocity equation on page 33 of this document).

\[
\omega = 2\pi f \quad \text{(angular frequency of charge currents on TSO great circle)}
\]

\[
f = \frac{c}{2\pi r} \quad \text{(orbit frequency)}
\]

\[
r = na_0 \quad \text{(radius* of TSO, GUTCP Eq. (I.107))}
\]

\[
n = \alpha \quad \text{(orbit state)}
\]

\[
\alpha = \frac{e^2}{\hbar c(4\pi\varepsilon_0)} \quad \text{(fine structure constant)}
\]

\[
a_0 = \frac{\hbar^2 (4\pi\varepsilon_0)}{m_0e^2} \quad \text{(Bohr radius)}
\]

\[
\hbar = \text{reduced Planck's constant}
\]

*Note: This presentation mostly uses the Bohr radius \( a_0 \) which does not include the “reduced mass” concept since the equations are used to calculate quantities for the TSO. The TSO does not have a proton at the center and therefore there is no reduced mass correction.
This collision causes a volume of space equal to the volume of the TSO to resonate between electric and magnetic energy with a total energy of 1.022 MeV. One half of the volume resonates with 510998.896 eV and creates the electron and the other half resonates with 510998.896 eV and creates the positron.

\[ = m_0 c^2 = 510998.896 \text{eV} \]

Details:
The collision of the photon and the nucleus (such as a proton) results in a volume of space the size of the transition state orbitsphere (TSO) ringing at its resonant frequency with a magnetic and electric oscillation similar to a resonant LC electrical circuit. This resonant frequency is calculated using ½ of the inductance \( L \) and ½ of the capacitance \( C \) of a conducting sphere (the bound electron is a conducting sphere) that is the size of the TSO. Multiplying this resonant angular frequency by \( \hbar \) (hbar) gives 510998.896 eV.

Note: I am unsure about the method of inserting the factor of ½ into the capacitance and inductance formulas but it is clear that the factors need to be there. GUTCP does not explicitly state that ½ of the capacitance and ½ of the inductance of a conducting sphere is used. But the ½ factor seems to be implied by the equations in GUTCP assuming a sphere is the proper geometry that should be used.
I think this $\frac{1}{2}$ factor comes from the fact that $\frac{1}{2}$ the volume of the sphere resonates for the electron and (independently) the other $\frac{1}{2}$ resonates for the positron. I found the following statement in the Leptons section of GUTCP (Chapter 36) which might back up my reasoning:

*Because two magnetic moments are produced, the magnetic energy (and corresponding photon frequency) in the proper frame is two times that of the electron frame. Thus, the electron time is corrected by a factor of two relative to the proper time.* [emphasis added]

The quote above can be found just above GUTCP Eq. (36.7) in the chapter on Leptons.

The standard equation for the capacitance of an isolated sphere of radius $r$ is

$$C = 4\pi \varepsilon_0 r$$  
(the field lines extend out to infinity)

Then insert the TSO radius

$$r = n a_0 = \alpha a_0$$  
(radius of TSO)

so the capacitance becomes

$$C = 4\pi \varepsilon_0 \alpha a_0$$

GUTCP then includes a $\frac{1}{2\pi}$ relativistic factor (see Figure 1.33 in GUTCP) and also includes an *unstated* factor of $\frac{1}{2}$. I am *speculating* that the $\frac{1}{2}$ factor comes from the fact that $\frac{1}{2}$ of the TSO volume resonates due to the electron (from one 511 keV photon) and the other $\frac{1}{2}$ of the volume resonates due to the positron (from the other 511 keV photon). The capacitance then becomes

(capacitance)  \[ C = \left(\frac{1}{2\pi}\right)\left(\frac{1}{2}\right) 4\pi \varepsilon_0 \alpha a_0 = \alpha a_0 \varepsilon_0 \] 

See GUTCP Eq. (29.17)
GUTCP uses a similar looking formula for the inductance of the sphere but swaps the permittivity \( \varepsilon_0 \) factor for the permeability factor \( \mu_0 \). The inductance of the sphere is

\[
\text{inductance} \quad L = \alpha a_0 \mu_0 \quad \text{GUTCP Eq. (29.18)}
\]

I was not able to confirm this equation for the inductance of a sphere but GUTCP must be correct since the characteristic impedance of free space is equal to:

\[
\sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7303 \text{ ohms} \quad \text{GUTCP Eq. (29.16)}
\]

Since there are no factors in front of the permittivity constant \( \varepsilon_0 \) or the permeability constant \( \mu_0 \) in the equation for the impedance of free space (GUTCP Eq. (29.16)), the conclusion is that those constants are equal and cancel out. This indicates that GUTCP has the correct equation for the inductance \( L \) assuming the capacitance equation for \( C \) is correct.

Using these equations for capacitance and inductance, the electric and magnetic resonance of a volume of space that equals \( \frac{1}{2} \) the volume of the TSO is

\[
\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\alpha a_0 \varepsilon_0 \alpha a_0 \mu_0}} = \frac{m_0 c^2}{\hbar} = 7.7634408 \times 10^{20} \text{ rad/s} \quad \text{GUTCP Eq. (29.19)}
\]

Multiplying this frequency by \( \hbar \) (the reduced Planck constant) gives 510998.896 eV.
Where:
\[ \varepsilon_0 = \text{permittivity of free space} \]
\[ \mu_0 = \text{permeability of free space} \]
\[ a_0 = \frac{\hbar^2 (4\pi\varepsilon_0)}{m_0e^2} = \text{the Bohr radius} \]
\[ c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \text{speed of light} \]
\[ \alpha = \frac{e^2}{\hbar c (4\pi\varepsilon_0)} = \text{fine structure constant} \]

The resonant frequency for the TSO matches the frequency of a 510998.896 eV photon:

\[ \omega = \frac{E}{\hbar} = \frac{m_0c^2}{\hbar} = 7.7634408 \times 10^{20} \text{ rad/sec} \]

The calculations show that the resonant frequency of a volume of space equal to $\frac{1}{2}$ the TSO volume matches the frequency of a photon having 510998.896 eV. Therefore Table 1 lists this as “Resonant Energy (photon energy equivalent)”. 

The electric / magnetic oscillation of free space during pair production is similar to a parallel LC circuit where the resonant frequency is a function of the capacitance and inductance:

\[ \omega = \frac{1}{\sqrt{LC}} \]
The electron orbit velocity slows down to zero as it ionizes to infinity. The magnetic field energy also drops to zero. The change in electric potential energy between the positron and the electron in the TSO as each ionizes to infinity is given in GUTCP using an equation that matches the form used for the hydrogen atom with the trapped photon (and \(1/n\) electric field factor) included*. Even though the GUTCP derivation for the electric potential energy is hard to follow, (see GUTCP Eq. 29.1 through Eq. 29.10) it is clear that the electric potential energy for the hydrogen atom approaches 510998.896 eV as the radius approaches \(n = \alpha\) (alpha = 1/137.035999). The electric potential energy equation for the hydrogen atom has to be multiplied by two to get 1.022 MeV because the electric potential energy calculation for one TSO sized hydrogen atom amounts to 510998.896 eV.

\[
\Delta PE = \frac{e^2}{n(4\pi\varepsilon_0)r_2} - \frac{e^2}{n(4\pi\varepsilon_0)r_1} = -m_0c^2 = -510998.896 \text{ eV}
\]

Evaluated at: \(r_1 = \text{radius of TSO}\) \(r_2 = \text{infinity}\)

*The TSO doesn’t have a trapped photon but the hydrogen atom does. See the discussion near GUTCP Eq. (29.10) for the TSO electric potential energy equation where it includes the multiplication factor \(\alpha\) (alpha = 1/137.035999) and states it "arises from Gauss' law surface integral and the relativistic invariance of charge".

\[
3. \quad \varepsilon_0 = \pi
\]

\[
\text{see GUTCP Eq. (29.3)}
\]
The equation above matches the electric potential energy equation for the hydrogen atom. For this document, it will be easier to derive the equation for the hydrogen atom instead of the TSO and the reader can go to GUTCP for the TSO version which can be seen starting at GUTCP Eq. (29.1) and ending with GUTCP Eq. (29.10).

The derivation starts with the equation for change in electric potential energy between the proton and the electron in the hydrogen atom evaluated at $r_1$ and $r_2$ where $r_1$ will be set to the TSO radius and $r_2$ will be set to infinity

$$\Delta PE = -\frac{e^2}{\alpha^2 (4\pi\varepsilon_0) a_0} = -510998.896 \text{ eV} \quad \text{(GUTCP Eq. (29.10))}$$

Details: The electron and the positron in the transition state orbitsphere (TSO) ionize to infinity (i.e. become a free electron and a free positron) with the energy to separate them equal to: $2 \times 510998.896 \text{ eV} = 1.022 \text{ MeV}$ total. GUTCP derives the electric potential energy for the electron in the TSO:

$$\Delta PE = -\frac{e^2}{\alpha^2 (4\pi\varepsilon_0) a_0} = -510998.896 \text{ eV} \quad \text{(GUTCP Eq. (29.10))}$$

The equation above matches the electric potential energy equation for the hydrogen atom. For this document, it will be easier to derive the equation for the hydrogen atom instead of the TSO and the reader can go to GUTCP for the TSO version which can be seen starting at GUTCP Eq. (29.1) and ending with GUTCP Eq. (29.10).

The derivation starts with the equation for change in electric potential energy between the proton and the electron in the hydrogen atom evaluated at $r_1$ and $r_2$ where $r_1$ will be set to the TSO radius and $r_2$ will be set to infinity

$$\Delta PE = \frac{e^2}{(4\pi\varepsilon_0) r_2} - \frac{e^2}{(4\pi\varepsilon_0) r_1} \quad \text{(Eq. (A))}$$

In the electron orbitsphere, the charge of the electron is the elementary charge $e$ but the electric field that the electron experiences changes with $1/n$ for each orbit state $n$ due to the “trapped photon”.

29
As a result, in Eq. (A) on the previous page, $e^2$ becomes $\frac{e^2}{n}$ and Eq. (A) becomes

$$\Delta PE = \frac{e^2}{n(4\pi\varepsilon_0)r_2} - \frac{e^2}{n(4\pi\varepsilon_0)r_1}$$

(Eq. (B))

Inserting the 5 equations below into Eq. (B) gives $-m_0c^2$ or -510998.896 eV which is the negative of the rest mass of the electron. The negative sign only indicates that it needs energy input to move the electron from the TSO orbit out to infinity.

<table>
<thead>
<tr>
<th>$r_1 = na_0$ (radius of transition state orbitsphere)</th>
<th>$n = \alpha$ (orbit state)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_2 = \text{infinity}$</td>
<td>$a_0 = \frac{\hbar^2 (4\pi\varepsilon_0)}{m_0e^2}$ (Bohr radius)</td>
</tr>
<tr>
<td>$\alpha = \frac{e^2}{\hbar c (4\pi\varepsilon_0)}$ (fine structure constant)</td>
<td></td>
</tr>
</tbody>
</table>
The resonant magnetic and electric oscillation of the TSO transitions into charge currents that travel on an infinite number of circular paths at the speed of light \( c \) on the surface of the transition state orbitsphere (TSO). Collectively they cover the entire surface of the electron orbitsphere and travel on great circles (a great circle is the largest circle that can be drawn on a sphere). The charge currents create a magnetic field energy and the total magnetic field energy is equal to two multiplied by 510998.896 eV which equals 1.022 MeV (for the electron and the positron). The equation for this magnetic energy is derived in GUTCP. Insert the following equations on the next page into the equation above for proof.

\[
E_{\text{mag}} = \frac{\pi \mu_0 e^2 \hbar^2}{\alpha (2\pi m_0)^2 r^3} = m_0 c^2 = 510998.896 \text{ eV}
\]

See GUTCP Eq. (1.162, 29.14 and 32.32b)

Details:
The resonant magnetic and electric oscillation of the TSO transitions into charge currents that travel on an infinite number of circular paths at the speed of light \( c \) on the surface of the transition state orbitsphere (TSO). Collectively they cover the entire surface of the electron orbitsphere and travel on great circles (a great circle is the largest circle that can be drawn on a sphere). The charge currents create a magnetic field energy and the total magnetic field energy is equal to two multiplied by 510998.896 eV which equals 1.022 MeV (for the electron and the positron). The equation for this magnetic energy is derived in GUTCP. Insert the following equations on the next page into the equation above for proof.

*Question: Do these two magnetic fields cancel each other?? Are they sinusoidally out of phase with each other by 90 or 180 degrees??.
In other words:
The energy stored in the magnetic field of the electron orbitsphere equals 510998.896 eV if the radius is equal to the transition state orbitsphere (TSO).

\[ r = n\alpha_0 \quad \text{(radius of Transition State Orbitsphere)} \]

\[ n = \alpha \quad \text{(orbit state)} \]

\[ a_0 = \frac{\hbar^2 (4\pi\varepsilon_0)}{m_0e^2} \quad \text{(Bohr radius)} \]

\[ \alpha = \frac{\mu_0e^2c}{2\hbar} \quad \text{(fine structure constant)} \]

\[ c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \quad \text{(speed of light)} \]

\[ \hbar \quad \text{= reduced Planck’s constant} \]

In other words:
The energy stored in the magnetic field of the electron orbitsphere equals 510998.896 eV if the radius is equal to the transition state orbitsphere (TSO).

TSO radius = \[ r = n\alpha_0 = \alpha a_0 = .00386159 \text{ Angstroms} \]
Orbitsphere surface current velocity:

In the TSO, the velocity of the electron charge currents as they orbit around the proton on the great circles is equal to the speed of light $c$. The velocity equation is derived in The Grand Unified Theory of Classical Physics and it applies at all orbit states $n$. It should be noted that the velocity of the electron in the GUTCP model matches the velocity of the electron in the Bohr Model for the same principal quantum number $n$ but the angular frequency is not the same because the radius is different (additionally, the Bohr Model does not allow fractional principal quantum numbers $n$).

Proof is seen by inserting following equations into the equation above:

\[
\alpha = \frac{e^2}{\hbar c (4\pi\varepsilon_0)} \quad \text{(fine structure constant)}
\]

\[
\mathbf{c} = \text{speed of light}
\]

\[
\begin{align*}
\text{TSO surface current velocity} & = v = \frac{e^2}{n\hbar(4\pi\varepsilon_0)} = c \\
\end{align*}
\]

Equation is derived in GUTCP and similar version shown in GUTCP Eq. (1.35). Also matches electron velocity equation for Bohr Model.

This can also be derived by starting with GUTCP Eq. (I.61) (in the Introduction) and inserting the TSO radius $r = \alpha a_0$. 

\[
\text{Equation is derived in GUTCP and similar version shown in GUTCP Eq. (1.35). Also matches electron velocity equation for Bohr Model.}
\]

\[
\text{This can also be derived by starting with GUTCP Eq. (I.61) (in the Introduction) and inserting the TSO radius $r = \alpha a_0$.}
\]
In GUTCP, the Mass/Spacetime Metric Energy is equal to

5. Mass/Spacetime Metric Energy
\[ \alpha^{-1} \frac{\mu e^2 c^2}{2h} \sqrt{\frac{G m_0}{\hat{\lambda}_C}} \sqrt{\frac{\hbar c}{G}} = m_0 c^2 \]

GUTCP writes a similar equation in Eq. (32.48b) and includes the “sec” which seems to be a correction due to the Gravitational constant being off by 0.25%.

\[ E = \frac{\alpha h}{1 \text{ sec}} \sqrt{\frac{\hbar c^2}{2Gm}} = m_0 c^2 \quad \text{(GUTCP Eq. (32.48b))} \]

Proof is seen by inserting the following equations into the two equations above:

\[ \hat{\lambda}_C = r \quad \text{Compton wavelength bar} = \text{radius of TSO} \]

\[ r = n a_0 \quad \text{(TSO radius)} \]

\[ n = \alpha \quad \text{(orbit state)} \]

\[ G = 6.67384 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \quad \text{(gravitational constant)} \]

\[ a_0 = \frac{\hbar^2 (4 \pi \varepsilon_0)}{m_0 e^2} \quad \text{(Bohr radius)} \]
In this document it was shown that 5 calculated energies related to the transition state orbitsphere are exactly equal to the rest mass of the electron \((510998.896 \text{ eV})\).

The 5 equations are very different classical equations:
1. Planck equation energy.
2. Resonant energy (photon energy equivalent).
3. Electric potential energy.
4. Magnetic energy.
5. Mass/Spacetime metric energy.

The equations are completely consistent with a classical approach to atomic theory that includes Special Relativity.

The fact that five calculated energies all equal exactly \(510998.896 \text{ eV}\) is a strong indication that Randell Mills has the correct model of the atom!
Relative size of hydrogen atoms and hydrinos.

Radius = \( r = 0.52946 \) Angstroms
Equal to Bohr radius \( a_0 \)

\[ n = 1 \]
(normal hydrogen)

\[ n = 1/2 \]
\[ n = 1/3 \]

Actually, transition state orbitsphere would be \( \frac{1}{4} \) of the size shown here.

Transition state orbitsphere (TSO)
Radius = \( r = 0.00386 \) Angstroms
Bohr Model
Bohr Model
Successful for simple hydrogen type (i.e. one electron) atoms.

Hydrogen photon emission

One electron atom photon emission
Bohr Model

- Uses **classical** electrodynamic equations. Does not include Einstein’s Special Relativity.
- Successfully calculates the light emission lines of hydrogen and other one electron atoms to an accuracy of 1 part in 2000 (or 1 part in 30,000 with reduced mass concept included).
- Derivation of energy equations starts by setting the centripetal acceleration force equal to the electrostatic force.

**Force due to Centripetal Acceleration (moving in a circular path):**

\[ \text{Force} = \frac{m v^2}{r} \]

**Force due to Electrostatic Attraction (from negative electron and positive proton):**

\[ \text{Force} = \frac{k_e e^2}{r^2} \]

**Centripetal Force = Electrostatic Force**

\[ \frac{m v^2}{r} = \frac{k_e e^2}{r^2} \]

**Bohr Model:** Forces on electron are balanced

Bohr Model: Postulates used in derivation of energy equations:

Angular momentum \( L \) for each orbit state \( n \):

\[ L = m v r = n \hbar \]

\( n = 1, 2, 3, 4 \ldots \) infinity

- \( L \) = angular momentum
- \( m \) = mass
- \( v \) = velocity of electron (point charge)
- \( r \) = radius of orbit
- \( \hbar \) = reduced Planck constant
- \( n \) = principal quantum number
### Classical Electrodynamics

- Forces on electric charges and currents described using simple equations.
- Developed in the 1800’s by Maxwell and others.
- Includes Maxwell’s Equations, Lorentz Force, Coulomb Force, Newton’s Equations etc.
- Does not include quantum theory.

### Final Bohr Model equations for hydrogen.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Potential Energy} = -\frac{e^2}{(4\pi\varepsilon_0) n^2 a_0} )</td>
<td></td>
</tr>
<tr>
<td>( \text{Kinetic Energy} = \frac{e^2}{2(4\pi\varepsilon_0) n^2 a_0} )</td>
<td></td>
</tr>
<tr>
<td>( \text{Total Energy} = -\frac{e^2}{2(4\pi\varepsilon_0) n^2 a_0} )</td>
<td>Total Energy equation is used to calculate energy of an emitted photon when the electron drops from one orbit state ( n ) to a lower orbit state ( n ) (where ( n ) is the principal quantum number).</td>
</tr>
<tr>
<td>( \text{Binding Energy} = E_B = \frac{e^2}{2(4\pi\varepsilon_0) n^2 a_0} )</td>
<td></td>
</tr>
<tr>
<td>( \text{electron orbit velocity} = v = \frac{e^2}{n\hbar(4\pi\varepsilon_0)} )</td>
<td></td>
</tr>
<tr>
<td>( \text{electron radius} = r = n^2 a_0 )</td>
<td></td>
</tr>
</tbody>
</table>
Bohr Model
Successful in some areas...
Hydrogen emission lines calculated with 0.05% accuracy (i.e. 1 part in 2000).

Inverse of photon wavelength equals:

\[ \frac{1}{\lambda} = \frac{R_E}{hc} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

where

Rydberg Energy = \( R_E = \frac{4e^4m}{8\varepsilon_0^2h^2} \)

Hydrogen emission lines

Light Emissions From Hydrogen Atom. Measured Values and Calculated Values Using Bohr Model. Based On Standard Accepted Theory.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ( \rightarrow ) 1</td>
<td>121.50</td>
<td>121.50</td>
<td>Lyman Series</td>
</tr>
<tr>
<td>3 ( \rightarrow ) 1</td>
<td>102.52</td>
<td>102.5</td>
<td></td>
</tr>
<tr>
<td>4 ( \rightarrow ) 1</td>
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<td>97.2</td>
<td></td>
</tr>
<tr>
<td>5 ( \rightarrow ) 1</td>
<td>94.92</td>
<td>94.9</td>
<td></td>
</tr>
<tr>
<td>6 ( \rightarrow ) 1</td>
<td>93.73</td>
<td>93.7</td>
<td></td>
</tr>
<tr>
<td>7 ( \rightarrow ) 1</td>
<td>93.03</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>8 ( \rightarrow ) 1</td>
<td>92.57</td>
<td>92.6</td>
<td></td>
</tr>
<tr>
<td>9 ( \rightarrow ) 1</td>
<td>92.27</td>
<td>92.3</td>
<td></td>
</tr>
<tr>
<td>10 ( \rightarrow ) 1</td>
<td>92.05</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>11 ( \rightarrow ) 1</td>
<td>91.89</td>
<td>91.9</td>
<td></td>
</tr>
<tr>
<td>infinity ( \rightarrow ) 1</td>
<td>91.13</td>
<td>91.12</td>
<td></td>
</tr>
<tr>
<td>3 ( \rightarrow ) 2</td>
<td>656.11</td>
<td>656.3</td>
<td>Balmer Series</td>
</tr>
<tr>
<td>4 ( \rightarrow ) 2</td>
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<td>486.1</td>
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</tr>
<tr>
<td>5 ( \rightarrow ) 2</td>
<td>434.1</td>
<td>434.1</td>
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</tr>
<tr>
<td>6 ( \rightarrow ) 2</td>
<td>410.07</td>
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<tr>
<td>7 ( \rightarrow ) 2</td>
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<tr>
<td>8 ( \rightarrow ) 2</td>
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</tr>
<tr>
<td>9 ( \rightarrow ) 2</td>
<td>383.44</td>
<td>383.5</td>
<td></td>
</tr>
<tr>
<td>infinity ( \rightarrow ) 2</td>
<td>364.5</td>
<td>364.6</td>
<td></td>
</tr>
<tr>
<td>4 ( \rightarrow ) 3</td>
<td>1874.6</td>
<td>1870</td>
<td>Paschen Series</td>
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<td>1280</td>
<td></td>
</tr>
<tr>
<td>6 ( \rightarrow ) 3</td>
<td>1093.5</td>
<td>1090</td>
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<tr>
<td>7 ( \rightarrow ) 3</td>
<td>1004.7</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>8 ( \rightarrow ) 3</td>
<td>954.3</td>
<td>954</td>
<td></td>
</tr>
<tr>
<td>infinity ( \rightarrow ) 3</td>
<td>820.1</td>
<td>820</td>
<td></td>
</tr>
</tbody>
</table>

The spectral series of hydrogen, on a logarithmic scale. Photon wavelength
Bohr Model of the atom fails to predict other atomic quantities (including quantities for 2 electron atoms).

As a result, **Standard Quantum Mechanics** was created and it too had problems predicting many quantities.

But **GUTCP** succeeds everywhere including where the previous two models (Bohr Model and Standard Quantum Mechanics) fail.
Randell Mills’s
The Grand Unified Theory of Classical Physics (GUTCP)
GUTCP is similar to Bohr Model for the hydrogen atom.

**Except that GUTCP:**

- Includes a “trapped photon” that results in an the electron experiencing an electric charge equal to $\frac{e}{n}$ between the electron and the proton.
- Has a radius $r = n a_0$ (where $n = \text{orbit state and } a_0 = \text{Bohr radius}$).
- Electron is made of infinitesimal charges and masses that orbit on a sphere.
- Includes relativistic effects (i.e. Special Relativity).
- Allows integer and fractional principal orbit states

\[ n = 1, 2, 3, 4 \ldots \text{infinity} \]

\[ n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{p} \text{ where } p \leq 137 \]
GUTCP

- Uses **classical** electrodynamic equations. Includes Einstein’s Special Relativity.
- Matches Bohr model in successfully calculating the emission lines of hydrogen and other one electron atoms (though GUTCP successfully goes much further than this).
- Derivation of energy equations starts by setting the centripetal acceleration force equal to the electrostatic force (same as Bohr model).
- Postulates used in derivation of energy equations are different than the Bohr Model.

**GUTCP: Postulates used in derivation of energy equations:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular Momentum</td>
<td>$L = mvr = \frac{\hbar}{n}$</td>
</tr>
<tr>
<td>Electric Charge Experiance by Electron</td>
<td>$\frac{e}{n}$</td>
</tr>
</tbody>
</table>

In the Bohr Model, this is just $e$, the charge of the proton.

$n = \left\{ \begin{array}{l} n = 1, 2, 3, 4 \ldots \infty \\ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{p} \end{array} \right.$ and $p \leq 137$

Where $L$ equals angular momentum ($L = \text{mass}*\text{velocity} * \text{radius}$).
### Classical Electrodynamics

- Forces on electric charges and currents described using simple equations.
- Developed in the 1800’s by Maxwell and others.
- Includes Maxwell’s Equations, Lorentz Force, Coulomb Force, Newton’s Equations etc.
- Does not include quantum theory.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{e^2}{(4\pi\varepsilon_0)n^2a_0}$</td>
<td>Potential Energy</td>
</tr>
<tr>
<td>$\frac{e^2}{2(4\pi\varepsilon_0)n^2a_0}$</td>
<td>Kinetic Energy</td>
</tr>
<tr>
<td>$\frac{e^2}{2(4\pi\varepsilon_0)n^2a_0}$</td>
<td>Total Energy</td>
</tr>
<tr>
<td>$\frac{e^2}{2(4\pi\varepsilon_0)n^2a_0}$</td>
<td>Binding Energy</td>
</tr>
<tr>
<td>$\frac{e^2}{n\hbar(4\pi\varepsilon_0)}$</td>
<td>Electron orbit velocity</td>
</tr>
<tr>
<td>$na_0$</td>
<td>Electron radius</td>
</tr>
</tbody>
</table>

#### Final GUTCP equations for hydrogen.

Total Energy equation is used to calculate energy of an emitted photon when the electron drops from one orbit state $n$ to a lower orbit state $m$ (where $n$ is the principal quantum number).

Note that all equations listed here match the Bohr Model equations except for electron radius.
Comparison of Postulates used by Bohr Model and GUTCP during derivation of energy equations.

<table>
<thead>
<tr>
<th></th>
<th>Electric field factor between electron and proton</th>
<th>Angular momentum for each orbit state ( n ) ((L = \text{mass} \times \text{velocity} \times \text{radius}))</th>
<th>Principal quantum number ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bohr Model</strong></td>
<td>( 1 )</td>
<td>( n \hbar )</td>
<td>( n = 1, 2, 3, 4 \ldots \text{infinity} )</td>
</tr>
<tr>
<td><strong>GUTCP</strong></td>
<td>( \frac{1}{n} ) ( (\text{due to trapped photon}) )</td>
<td>( \hbar ) ( (\text{reduced Planck constant}) )</td>
<td>( n= \left{ \begin{array}{c} 1, 2, 3 \ldots \text{infinity} \ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{p} \end{array} \right} \text{ and } p \leq 137 )</td>
</tr>
</tbody>
</table>

Side Note: The postulate for angular momentum used in the Bohr Model can alternatively be that the circumference of the electron orbit is equal to the principal quantum number \( n \) multiplied by the electron’s de Broglie wavelength while in GUTCP it can be that the circumference of the orbit circle is equal to one de Broglie wavelength at all orbit states \( n \). Using this wavelength postulate gives the same result as the angular momentum postulate in this column.
GUTCP model of the atom

Successfully calculates many quantities for the atom such as:

- spectral line emission from the hydrogen atom (same as Bohr Model)
- magnetic moment
- g factor
- spin
- orbital angular momentum
- spin orbit splitting
- Lamb Shift
- fine and hyperfine structure line splitting
- lifetime of excited states
- selection rules
- multi-electron atoms
- ionization energies for atoms with 1 to 20 electrons

And more using much simpler equations compared to Standard Quantum Mechanics.
GUTCP model of the hydrogen atom:

1. Positive charged nucleus at the center

2. Negative charged electron in the form of a thin spherical shell called an electron orbitsphere.

The Electron Orbitsphere is the hydrogen atom when the proton is at the center.

Electron is an ultra-thin shell of electric charge and can be modeled as infinitesimal charges and masses orbiting on an infinite number of great circles.
Electron Orbitsphere

- Electron is a shell of electric charge surrounding the proton nucleus (or a positron).
- Can be modeled as an infinite number of infinitesimal sized charge currents that orbit on circular paths (“great circles”) around the proton (or around the positron).
- The transition state orbitsphere (TSO) is a special case of the electron orbitsphere with the positron (not the proton) providing the central electric field which gives the spherical shape (but see note at bottom of page 16 regarding this central attractive force).

Analogy used in the mathematical model:

Break an electron into an infinite number of infinitesimal pieces of mass and charge and have each piece orbit on an infinite number of “great circles” of a sphere.

In the model, each infinitesimal charge and mass is in force balance.

Each infinitesimal point charge and point mass orbits with the same velocity \( \mathbf{v} \) and angular frequency \( \omega \) on each great circle.
Bohr Model
Point sized electrons orbit the proton like the planets orbit the Sun

Randell Mills Model
Electrons are concentric spherical shells of electric charge

Note: For hydrogen, the electron is only in one of the orbits shown in both models above.
## Differences between Bohr Model and GUTCP

<table>
<thead>
<tr>
<th>Item</th>
<th>Bohr Model</th>
<th>GUTCP</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius</td>
<td>$r = n^2 a_0$</td>
<td>$r = n a_0$</td>
<td>$a_0 = .0529$ nm</td>
</tr>
<tr>
<td>electric field experienced by electron (between proton and electron)</td>
<td>$e$</td>
<td>$\frac{e}{n}$</td>
<td>$e = \text{elementary charge}$</td>
</tr>
<tr>
<td>bound electron</td>
<td>orbiting point particle around proton</td>
<td>spherical shell of charge (infinitesimal orbiting point charges, masses)</td>
<td></td>
</tr>
<tr>
<td>trapped photon?</td>
<td>none</td>
<td>yes</td>
<td>contributes to electric field between electron and proton</td>
</tr>
<tr>
<td>orbit motion</td>
<td>planetary</td>
<td>orbit on “great circles”</td>
<td></td>
</tr>
<tr>
<td>angular momentum</td>
<td>equal to $n \hbar$</td>
<td>equal to $\hbar$ at all orbit states $n$</td>
<td>$\hbar = \text{reduced Planck’s constant}$</td>
</tr>
<tr>
<td>Principal quantum numbers allowed</td>
<td>$n = 1,2,3 \ldots$ infinity</td>
<td>$n = \left{ \begin{array}{ll} 1,2,3 \ldots \text{infinity} \ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{p} \end{array} \right.$ and $p \leq 137$</td>
<td></td>
</tr>
<tr>
<td>Includes general relativity (time dilation, length contraction)</td>
<td>No</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>
Appendix A

Overview of Relativistic Equations for Hydrogen
Consider that in fact, the mathematics of the three theories of Bohr, Schrödinger, and presently CP* converge to Eq. (I.1) as the principal energy levels of the hydrogen atom.

\[
E_n = -\frac{e^2}{n^2 8\pi \varepsilon_0 a_H} = -\frac{13.598 \text{ eV}}{n^2}
\]

\( n = 1, 2, 3, \ldots \)

GUTCP Eq. (I.1)

Where \( a_H \) is the Bohr radius for the hydrogen atom (52.947 pm), \( e \) is the magnitude of the charge of the electron, and \( \varepsilon_0 \) is the vacuum permittivity. The theories of Bohr and Schrödinger depend on specific postulates to yield Eq. (I.1). A mathematical relationship exists between the theories of Bohr and Schrödinger with respect to CP that involves these postulates.

*CP is the acronym for Classical Physics and represents the same thing as GUTCP.
And later in chapter 1, GUTCP shows Eq. (1.293) and Eq. (1.294) which are equations for the binding energy $E_b$ of one electron atoms (i.e. any number of protons with just one electron) and compares it with measured data for hydrogen through uranium. Compared to the measured value, GUTCP Eq. (1.293) shows an error of .0002% for hydrogen and a maximum error of .32% for uranium. This is strong proof that one electron atoms behave in a manner described by GUTCP.

When GUTCP Eq. (1.293) is applied to hydrogen, it only applies at $n = 1$ and does not include orbit state $n$ for fractional or excited energy states. Appendixes A, B and C in this document derive and discuss Eqs. (B.40) and (C.52) which includes orbit state $n$ so that it can be used for fractional and excited states of hydrogen.

Eqs. (B.40) and (C.52) in Appendixes A, B and C (in this document) and the resulting graphs show the following:

- Eq. (C.52) can be used to calculate the photon energy emitted for any excited hydrogen principal orbit state transition with a maximum error of .05% (i.e. 1 part in 2000) from the measured value.
- Eq. (B.40) is virtually the same as Eq. (C.52) but ignores spin-nuclear interaction and perfectly intersects the rest mass of the electron (51098.896 eV) at orbit state $n = \alpha$ where $\alpha = 1/137.03599$ and is the fine structure constant.
- GUTCP Eq. (1.293) and Eqs. (B.40) and (C.52) include the Lorentz relativistic factor and thus are based on Einstein’s theory of relativity:

$$
\text{Lorentz relativistic factor} = \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}
$$
GUTCP gives the binding energy for **one electron atoms** (i.e. having any number of protons and one electron) as the following:

\[
E_B = \frac{(\alpha Z)^2 m_e c^2}{\sqrt{1 - \left(\frac{\alpha Z}{1 + \frac{m_e}{2m_p A}}\right)^2} + \frac{m_e}{m_p A}} - \frac{m_e c^2}{\sqrt{1 - \left(\frac{\alpha Z}{1 + \frac{m_e}{2m_p A}}\right)^2}} - 1
\]

**GUTCP Eq. (1.293)**

Appendix C (in this document) shows the derivation of the binding energy equation, \(E_B\), for the **hydrogen atom** based on identical methods from GUTCP’s derivation of GUTCP Eq. (1.293) (shown above) and the result is:

\[
E_B = \frac{\alpha^2 m_e c^2}{n^2 \sqrt{1 - \left(\frac{\alpha}{n \left(1 + \frac{2m_{\text{orbit}}}{2m_p}\right)}\right)^2} + \frac{m_e}{m_p}} - \frac{m_e c^2}{\sqrt{1 - \left(\frac{\alpha}{n \left(1 + \frac{2m_{\text{orbit}}}{2m_p}\right)}\right)^2}} - 1
\]

**Eq. (C.52)**

Eq. (C.52) is derived in Appendix C.
**In GUTCP, the spin-nuclear interaction is related to the reduced mass effect and GUTCP states that in the cases that the spin-nuclear interaction is negligible for one electron atoms, GUTCP Eq. (1.293) reduces to GUTCP Eq. (1.294):**

\[
E_B = m_{e_0} c^2 \left(1 - \sqrt{1-(\alpha Z)^2}\right)
\]  
**GUTCP Eq. (1.294)**

Similarly, in the cases where the spin-nuclear interaction is considered negligible for the hydrogen atom, Eq. (C.52) reduces to Eq. (B.40):  

\[
E_B = m_{e_0} c^2 \left(1 - \sqrt{1-\left(\frac{\alpha}{n}\right)^2}\right)
\]  
**(B.40)**

Eq. (B.40) is derived in Appendix B.
Figure (A1) shows the ionization energy for one electron atoms (i.e. any number of protons but just one electron) and compares the calculated value based on GUTCP Eq. (1.293) to the measured value. The maximum error is .32% for uranium. Measured values are from GUTCP Table 1.5 with the reduced mass factor included (though the measured values from GUTCP Table 1.5 were adjusted to include the reduced mass factor for increased accuracy).

In the graphs above: Difference(%) = 100%[Measured – GUTCP Eq. (1.293)]/ GUTCP Eq. (1.293)
Comparison of theoretical and measured ionization energy using GUTCP Eq. (1.293). Theoretical matches measured value to within:
- 0.06% for $Z < 35$
- 0.3% for $Z = 92$ (Uranium)

Image source: Figure 1.35 in The Grand Unified Theory of Classical Physics, R. Mills.
Table 1.5. Relativistic ionization energies for some one-electron atoms.

<table>
<thead>
<tr>
<th>One Atom</th>
<th>Z (Eq. (1.288))</th>
<th>β</th>
<th>Theoretical Ionization Energies (eV) (Eq. (1.293))</th>
<th>Experimental Ionization Energies (eV)</th>
<th>Relative Difference between Experimental and Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>0.00730</td>
<td>13.59847</td>
<td>13.59844</td>
<td>-0.000002</td>
</tr>
<tr>
<td>He⁺</td>
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<td>0.01459</td>
<td>154.41826</td>
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<td>-0.000009</td>
</tr>
<tr>
<td>Li²⁺</td>
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<td>0.02189</td>
<td>122.45637</td>
<td>122.45429</td>
<td>-0.00017</td>
</tr>
<tr>
<td>Be³⁺</td>
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<td>0.02919</td>
<td>217.72427</td>
<td>217.71865</td>
<td>-0.00026</td>
</tr>
<tr>
<td>B⁴⁺</td>
<td>5</td>
<td>0.03649</td>
<td>340.23871</td>
<td>340.2258</td>
<td>-0.00038</td>
</tr>
<tr>
<td>C⁵⁺</td>
<td>6</td>
<td>0.04378</td>
<td>490.01759</td>
<td>489.99334</td>
<td>-0.00049</td>
</tr>
<tr>
<td>N⁶⁺</td>
<td>7</td>
<td>0.05108</td>
<td>667.08834</td>
<td>667.046</td>
<td>-0.00063</td>
</tr>
<tr>
<td>O⁷⁺</td>
<td>8</td>
<td>0.05838</td>
<td>871.47768</td>
<td>871.4101</td>
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<tr>
<td>F⁸⁺</td>
<td>9</td>
<td>0.06568</td>
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<td>1103.1176</td>
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<td>Ne⁹⁺</td>
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<td>-0.00109</td>
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<td>Na¹⁰⁺</td>
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<td>1648.702</td>
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</tr>
<tr>
<td>Mg¹¹⁺</td>
<td>12</td>
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<td>1962.665</td>
<td>-0.00013</td>
</tr>
<tr>
<td>Al¹²⁺</td>
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<td>2304.141</td>
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</tr>
<tr>
<td>Si¹³⁺</td>
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<td>0.10216</td>
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<td>2673.182</td>
<td>-0.000178</td>
</tr>
<tr>
<td>P¹⁴⁺</td>
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<tr>
<td>S¹⁵⁺</td>
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<td>-0.000217</td>
</tr>
<tr>
<td>Cl¹⁶⁺</td>
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<td>0.12405</td>
<td>3947.228</td>
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<tr>
<td>Ar¹⁷⁺</td>
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<td>Kr¹⁸⁺</td>
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<tr>
<td>Ca¹⁹⁺</td>
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<td>5471.494</td>
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</tr>
<tr>
<td>Sc²⁰⁺</td>
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<td>6033.712</td>
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</tr>
<tr>
<td>Ti²¹⁺</td>
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<td>0.16054</td>
<td>6628.064</td>
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<td>V²²⁺</td>
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<td>0.16784</td>
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<td>7246.12</td>
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<tr>
<td>Cr²³⁺</td>
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<td>0.17514</td>
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<td>7894.81</td>
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<td>Mn²⁴⁺</td>
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<td>0.18243</td>
<td>8575.426</td>
<td>8571.94</td>
<td>-0.000407</td>
</tr>
<tr>
<td>Fe²⁵⁺</td>
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<td>0.18973</td>
<td>9281.650</td>
<td>9277.69</td>
<td>-0.000427</td>
</tr>
<tr>
<td>Co²⁶⁺</td>
<td>27</td>
<td>0.19703</td>
<td>10016.63</td>
<td>10012.12</td>
<td>-0.000450</td>
</tr>
<tr>
<td>Ni²⁷⁺</td>
<td>28</td>
<td>0.20432</td>
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<tr>
<td>Cu²⁸⁺</td>
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<td>0.21162</td>
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<tr>
<td>Zn²⁹⁺</td>
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<td>0.21892</td>
<td>12395.35</td>
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</tr>
</tbody>
</table>

Comparison of theoretical and measured ionization of one electron atoms using GUTCP Eq. (1.293).

Difference = 0.0126% (1 part in 8000)

Difference = 0.049% (1 part in 2000)
Figure (A3) shows the binding energy, $E_B$, for the electron in the hydrogen atom versus orbit state $n$ using Eq. (C.52) and compares it to the measured value for excited orbit states $n = 1$ through $n = 6$. The maximum error is $+.023\%$ at $n = 4$.

In the graph above: $\text{Difference(\%)} = \frac{100\%[\text{Measured} - \text{Eq. (C.52)}]}{\text{Eq. (C.52)}}$
<table>
<thead>
<tr>
<th>orbit state n</th>
<th>measured ionization energy (eV)</th>
<th>Eq. (C.52) (eV)</th>
<th>Eq. (B.40) (eV)</th>
<th>GUTCP I.1, with reduced mass factor included (eV)</th>
<th>GUTCP I.1, no reduced mass factor included (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>infinity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>0.3775418</td>
<td>0.377524915</td>
<td>0.377936017</td>
<td>0.377730159</td>
<td>0.377935877</td>
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<td>0.543635965</td>
<td>0.544227953</td>
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<td>2</td>
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</table>

Ground and Excited States

**Fractional Orbit States (selected values)**

<table>
<thead>
<tr>
<th>orbit state n</th>
<th>measured ionization energy (eV)</th>
<th>Eq. (C.52) (eV)</th>
<th>Eq. (B.40) (eV)</th>
<th>GUTCP I.1, with reduced mass factor included (eV)</th>
<th>GUTCP I.1, no reduced mass factor included (eV)</th>
</tr>
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<tbody>
<tr>
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<tr>
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<td>1103.251984</td>
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<tr>
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<tr>
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<td>1/137</td>
<td>-466527.178</td>
<td>499286.6979</td>
<td>255226.2245</td>
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<td></td>
</tr>
</tbody>
</table>

1/137.035999679

#DIV/0! 510998.8961 255360.3744 255499.448
Figure (A4) shows the hydrogen spectrum energy series based on Eq. (C.52) and compares it to the measured value for the Lyman Series, Balmer Series, Paschen Series, Bracket Series, Pfund Series and the Humphreys Series. The maximum error is +.05% for the Bracket Series for the transition $n = 8$ to $n = 4$.

In the graph above: Difference(%) = $100\% \frac{\text{Measured} - \text{Eq. (C.52)}}{\text{Eq. (C.52)}}$
Figure (A5) shows the Lyman series energies for the electron in the hydrogen atom based on different equations and compares the calculated value with the measured value. Eq. (C.52) has the least error (the least average percent difference). The values in this graph are also shown in Tables (A2) and (A3) on the next page.

In the graph above: Difference(%) = 100%[Measured – Eq. (X)]/ Eq. (X)
### Table A2. Hydrogen Lyman Series Energy

<table>
<thead>
<tr>
<th>initial orbit state n</th>
<th>measured Lyman series energy (eV)</th>
<th>Based on Eq. (C.52) (eV)</th>
<th>Based on Eq. (B.40) (eV)</th>
<th>Based on GUTCP I. 1, with reduced mass factor included (eV)</th>
<th>Based on GUTCP I. 1, no reduced mass factor included (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12.08053</td>
<td>12.08097</td>
<td>12.09413</td>
<td>12.08737</td>
<td>12.09395</td>
</tr>
<tr>
<td>2</td>
<td>10.19248</td>
<td>10.19334</td>
<td>10.20444</td>
<td>10.19871</td>
<td>10.20427</td>
</tr>
</tbody>
</table>

### Table A3. Hydrogen Lyman Series Energy: Percentage Difference Compared To Measured Value

<table>
<thead>
<tr>
<th>initial orbit state n</th>
<th>Difference Based on Eq. (C.52) (%)</th>
<th>Difference Based on Eq. (B.40) (%)</th>
<th>Difference Based on GUTCP I. 1, with reduced mass factor included (eV)</th>
<th>Difference Based on GUTCP I. 1, no reduced mass factor included (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>infinity</td>
<td>-0.005321672</td>
<td>-0.114101276</td>
<td>-0.058371183</td>
<td>-0.112771468</td>
</tr>
<tr>
<td>6</td>
<td>-0.005358369</td>
<td>-0.114138214</td>
<td>-0.058371183</td>
<td>-0.112771468</td>
</tr>
<tr>
<td>5</td>
<td>-0.005418385</td>
<td>-0.114198289</td>
<td>-0.058415029</td>
<td>-0.11281529</td>
</tr>
<tr>
<td>4</td>
<td>-0.006089694</td>
<td>-0.114869095</td>
<td>-0.059056273</td>
<td>-0.113456185</td>
</tr>
<tr>
<td>3</td>
<td>-0.00364054</td>
<td>-0.112423099</td>
<td>-0.05654423</td>
<td>-0.11094551</td>
</tr>
<tr>
<td>2</td>
<td>-0.008393695</td>
<td>-0.11717249</td>
<td>-0.061111487</td>
<td>-0.11551028</td>
</tr>
</tbody>
</table>

Tables (A2) and (A3) show the Lyman series energies for the electron in the hydrogen atom based on different equations and compares the calculated value with the measured value. Eq. (C.52) has the least absolute error with -.0083% for \( n = 2 \) to \( n = 1 \) transition (1 part in 12,000). Values are graphed in Figure (A5) on previous page. In the tables above: Difference(%) = \( 100\% \text{[Measured − Eq. (X)]/ Eq. (X)} \)
Appendix B

Relativistic Equations For Electron In Hydrogen Without The Reduced Mass Factor.
The derivation of the binding energy $E_B$ for excited and fractional orbits of hydrogen shown in this appendix does not include the reduced mass factor (note: the binding energy for the hydrogen atom is the same as ionization energy). The photon energy released by hydrogen for a transition between two orbit states can be calculated through subtraction of the binding energy for each of the two orbit states. GUTCP discusses the equations for the binding energy (and the principal energy levels) near GUTCP Eqs. (I.1), (I.45), (I.97), (I.118), (I.124) and (1.290).

The derivation in this appendix is based on a model of two charged point masses, the electron and the proton, where the electron orbits in a circle around the proton with the proton at the exact center of the electron’s orbit. This derivation assumes the proton is fixed in space and thus it does not include the reduced mass effect. In this derivation, assuming the proton is fixed in space decreases the complexity of the derivation and results in a simpler equation for binding energy $E_B$ as seen in the resulting equation (Eq. (B.40)).

Mills writes in GUTCP that the reduced mass effect is due to a magnetic interaction between the orbiting electron and the proton. Mills terms this force the $F_{mag}$ force and discusses it near GUTCP Eq. (I.92), (1.246) and (1.253).

The derived equation for the binding energy of hydrogen without the reduced mass factor (in this appendix) results in a calculated value of 13.606 eV for the ground orbit state. The derived equation for the binding energy of hydrogen with the reduced mass factor included (in Appendix C) results in a calculated value of 13.598 eV for the ground orbit state $n = 1$ which virtually, but not exactly, matches the measured ionization energy of 13.598 eV (using more decimal places would show a small difference).

The reduced mass factor is approximately equal to $1836.15/(1836.15+1) = .9994557$ where 1836.15 is the ratio of the mass of the proton to the mass of the electron. Multiplying 13.606 eV by .9994557 (the reduced mass factor) results in 13.598 eV.
The model is based on a negatively charged point mass $m_e$ with charge $-e$ (the electron) orbiting around the positively charged point mass $m_p$ with charge $+e$ (the proton) at a radius of $r$. The proton is assumed to be fixed in space and thus this derivation does not include the reduced mass effect.

Proton
$m_p = 1.67262 \times 10^{-27}$ kg
$+e = +1.60217 \times 10^{-19}$ C

$r_2 =$ radius to orbit center point
Proton is assumed to be fixed in space.

Electron
$m_e =$ mass varies as a function of orbit state $n$
$-e = -1.60217 \times 10^{-19}$ C

$r_1 =$ radius to orbit center point

Note: This model shown at left is just for setting up the math equations which match the spectroscopy data. See page 20 and 21 of this presentation that shows GUTCP’s spherical shell model.
The angular momentum $L$ of the electron/proton system is equal to $\hbar$ (the reduced Planck constant) at all orbit states. The angular momentum is equal to the moment of inertia $I$ multiplied by the angular velocity $\omega$:

$$L = I\omega = \hbar \quad \text{(B.1)}$$

The moment of inertia for a point particle orbiting around a center point is

$$I = mr^2$$

The moment of inertia for the electron having mass $m_e$ is then

$$I = m_e r^2 \quad \text{(B.2)}$$

Inserting moment of inertia equation (Eq. (B.2)) into the angular momentum equation (Eq. (B.1)) gives

$$L = I\omega = m_e r^2 \omega = \hbar \quad \text{(B.3)}$$

Angular velocity $\omega$ equals the orbit velocity $v$ divided by the radius $r$

$$\omega = \frac{v}{r} \quad \text{(B.4)}$$

Inserting angular velocity equation (Eq. (B.4)) into angular momentum equation (Eq. (B.3)) gives

$$L = \left( m_e r^2 \right) \frac{v}{r} = \hbar \quad \text{(B.5)}$$
Solving Eq. (B.5) for \( r \) gives

\[
    r = \frac{\hbar}{m_e v}
\]  

(B.6)

The radius equation above contains the orbit velocity \( v \) which is governed by the force balance equation between the proton and the orbiting electron where the centripetal acceleration force equals the electrostatic force:

\[
    \frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}
\]  

(B.7)

| centripetal force | electrostatic force |

But in GUTCP, there is the postulate that the charge that the electron experiences due to the proton is affected by orbit state \( n \) due to the trapped photon and this results in \( e^2 \) in the equation being replaced by \( \frac{e^2}{n} \) due to the trapped photon. So Eq. (B.7) becomes

\[
    \frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 n r^2}
\]  

(B.8)

The first postulate is introduced here in the force balance equation where the electrostatic force equation has \( e^2 \) replaced by \( e^2/n \) due to the trapped photon.

Solving Eq. (B.8) for velocity \( v \) gives

\[
    v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e n r}}
\]  

(B.9)
Inserting velocity equation (Eq. (B.9)) into the radius equation (Eq. (B.6)) gives

\[ r = \frac{\hbar}{m_e \sqrt{\frac{e^2}{4 \pi \varepsilon_0 m_e n \hbar}}} \]

Squaring both sides gives

\[ r^2 = \frac{\hbar^2 4 \pi \varepsilon_0 m_e n \hbar}{e^2 m_e^2} \]

Solving Eq. (B.10) for \( r \) gives

\[ r = \frac{\hbar^2 4 \pi \varepsilon_0 n}{e^2 m_e} \]

The bohr radius equals

\[ a_0 = \frac{\hbar^2 4 \pi \varepsilon_0}{m_e e^2} \]

Moving \( m_{e0} \) in Eq. (B.12) from the right side to the left side gives:

\[ a_0 m_{e_0} = \frac{\hbar^2 4 \pi \varepsilon_0}{e^2} \]
Inserting Eq. (B.13) into Eq. (B.11) gives

\[ r = \frac{a_0 m_{e0} n}{m_e} \quad \text{(B.14)} \]

\( m_{e0} \) in the equations above is a constant and equal to \( 9.109382 \times 10^{-31} \text{ kg} \) but \( m_e \) is the relativistic mass of the electron and varies as a function of velocity as given by the Lorentz transformation:

\[ m_e (v) = m_e = \frac{m_{e0}}{\sqrt{1 - \beta^2}} = \frac{m_{e0}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad \text{(B.15)} \]

An equation for the velocity of the electron in the hydrogen atom can be obtained from the condition that the electron/proton system orbits with an angular momentum of \( \hbar \) at all orbit states \( n \).

Angular momentum \( L = m_e v r = \hbar \) \quad \text{(B.16)}

Solving Eq. (B.16) for velocity gives

\[ v = \frac{\hbar}{m_e r} \quad \text{(B.17)} \]

Dividing both sides of Eq. (B.17) by \( c \) gives the relativistic parameter \( \beta \)

\[ \beta = \frac{v}{c} = \frac{\hbar}{m_e c r} \quad \text{(B.18)} \]

\( \text{(same as GUTCP Eq. (1.285))} \)
Inserting the radius equation (Eq. (B.14)) into the relativistic parameter equation (Eq. (B.18)) gives

\[
\beta = \frac{v}{c} = \frac{\hbar}{m_e c \left( \frac{n a_0 m_{e0}}{m_e} \right)} = \frac{\hbar}{c n a_0 m_{e0}}
\]  
(B.19)

(similar to GUTCP Eq. (1.283))

The equation for the fine structure constant \( \alpha \) is

\[
\alpha = \frac{\hbar}{m_{e0} c a_0}
\]  
(B.20)

Inserting the fine structure constant equation (Eq. (B.20)) into Eq. (B.19) gives

\[
\beta = \frac{v}{c} = \frac{\hbar}{c n a_0 m_{e0}} = \alpha \frac{1}{n}
\]  
(B.21)

Inserting Eq. (B.21) into the electron relativistic mass equation (Eq. (B.15)) gives

\[
m_e = \frac{m_{e0}}{\sqrt{1-\left( \frac{v}{c} \right)^2}} = \frac{m_{e0}}{\sqrt{1-\left( \frac{\alpha}{n} \right)^2}}
\]  
(B.22)

Inserting relativistic mass velocity equation (Eq. (B.22)) into the radius equation (Eq. (B.14)) gives

\[
r_n = \frac{a_0 m_{e0} n \sqrt{1-\left( \frac{\alpha}{n} \right)^2}}{m_{e0}} = a_0 n \sqrt{1-\left( \frac{\alpha}{n} \right)^2}
\]  
(B.23)
The total energy of the electron $E_{\text{Total}}$ is the sum of the potential energy $V_n$ and the kinetic energy $T_n$, which are each functions of orbit state $n$.

\[
E_{\text{Total}} = V_n + T_n \quad \text{(B.24)}
\]

(same as GUTCP Eq. (1.290))

The potential energy $V$ for the electron is

\[
V = \frac{-e^2}{(4\pi\varepsilon_0)r} \quad \text{(B.25)}
\]

(similar to GUTCP Eq. (1.261) except inserting $r_n$ for $r_1$ and setting $Z = 1$)

But in GUTCP, there is the postulate that the charge that the electron experiences due to the proton is affected by orbit state $n$ due to the trapped photon and this results in $e^2$ in Eq. (B.25) being replaced by $\frac{e^2}{n}$ so Eq. (B.25) becomes

\[
V_n = \frac{-e^2}{(4\pi\varepsilon_0)n r_n} \quad \text{(B.26)}
\]

The second postulate is introduced here in the potential energy equation where the electrostatic potential energy equation has $e^2$ replaced by $e^2/n$ due to the trapped photon.

Inserting the radius equation (Eq. (B.23)) into the potential energy equation (Eq. (B.26)) gives

\[
V_n = \frac{e^2}{(4\pi\varepsilon_0)n^2a_0\sqrt{1-(\frac{\alpha}{n})^2}} \quad \text{(B.27)}
\]
The bohr radius is
\[ a_0 = \frac{\hbar^2}{m_e e^2} 4 \pi \varepsilon_0 \]  
(B.28)

The fine structure constant is
\[ \alpha = \frac{\hbar}{m_e c a_0} \]  
(B.29)

Inserting the bohr radius equation (Eq. (B.28)) and the fine structure constant equation (Eq. (B.29)) into Eq. (B.27) and rearranging of the terms results in the potential energy equation:
\[ V_n = -\frac{m_e c^2 \alpha^2}{n^2 \sqrt{1-(\frac{\alpha}{n})^2}} \]  
(B.30)

The kinetic energy for the electron when it is traveling in a straight line with velocity \( v \) is
\[ T = m_e c^2 \left( \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1 \right) \]  
(B.31)  
(same as GUTCP Eq. (1.291))

Inserting the relativistic parameter \( \beta \) (Eq. (B.21)) into Eq. (B.31) gives the kinetic energy equation:
\[ T_n = m_e c^2 \left( \frac{1}{\sqrt{1-\left(\frac{\alpha}{n}\right)^2}} - 1 \right) \]  
(B.32)
Inserting the potential energy (Eq. (B.30)) and kinetic energy (Eq. (B.32)) equations into the total energy equation (Eq. (B.24)) gives

\[
E_{\text{Total}, n} = -\frac{m_e c^2 \alpha^2}{n^2 \sqrt{1 - \left(\frac{\alpha}{n}\right)^2}} + m_e c^2 \left(\frac{1}{\sqrt{1 - \left(\frac{\alpha}{n}\right)^2}} - 1\right)
\]

Next, the binding energy \(E_B\) is the energy required to ionize the electron (i.e. remove the bound electron from the atom out to a distance equal to infinity). The binding energy is the difference in total energy of the electron between final orbit state \(n_f = \infty\) and any specified initial orbit state \(n_i\).

\[
E_B = \Delta E_{\text{Total}} = E_{\text{Total}, f} - E_{\text{Total}, i}
\]

Where the subscript “\(f\)” is for the final orbit state and “\(i\)” is for the initial orbit state. At orbit state \(n = \infty\), the total energy, \(E_{\text{total}, f}\), is equal to zero and the binding energy (Eq. (B.34)) becomes:

\[
E_B = \Delta E_{\text{Total}} = 0 - E_{\text{Total}, i} = -E_{\text{Total}, i}
\]

Thus for a given orbit state \(n\), the binding energy \(E_B\) is the negative of the total energy.
The binding energy is then

\[ E_B = \frac{m_e c^2 \alpha^2}{n^2 \sqrt{1 - \left(\frac{\alpha}{n}\right)^2}} - m_e c^2 \left(\frac{1}{\sqrt{1 - \left(\frac{\alpha}{n}\right)^2}} - 1\right) \]  

(B.36)

The result above is important because it shows the full relativistic energy equation where the binding energy is equal to the kinetic energy subtracted from the negative of the potential energy*. Each of these three energies are graphed in Figure (B1).

*The potential energy is either zero or a negative number as a result of the standard convention of defining the potential energy as zero at infinite distance. Therefore the negative of the potential energy is always a positive number or zero. And the kinetic energy is always a positive number or zero.
If the kinetic energy term is expanded then Eq. (B.36) becomes

$$E_B = \frac{m_e c^2 \alpha^2}{n^2 \sqrt{1 - \left(\frac{\alpha}{n}\right)^2}} - \frac{m_e c^2}{\sqrt{1 - \left(\frac{\alpha}{n}\right)^2}} + m_e c^2$$

(B.37)

Which can be rewritten as

$$E_B = \frac{m_e c^2}{\sqrt{1 - \left(\frac{\alpha}{n}\right)}} \left(\frac{\alpha^2}{n^2} - 1\right) + m_e c^2$$

Multiplying all terms by -1 gives

$$-E_B = \frac{m_e c^2}{\sqrt{1 - \left(\frac{\alpha}{n}\right)}} \left(-\frac{\alpha^2}{n^2} + 1\right) - m_e c^2 = \frac{m_e c^2 \left(1 - \frac{\alpha^2}{n^2}\right)}{\sqrt{1 - \left(\frac{\alpha}{n}\right)}} - m_e c^2$$

taking the square root of the term in the numerator which is in brackets (with the blue arrow pointing at it) and then squaring it gives

$$-E_B = \frac{m_e c^2 \left(\sqrt{1 - \frac{\alpha^2}{n^2}}\right)^2}{\sqrt{1 - \left(\frac{\alpha}{n}\right)}} - m_e c^2 = \frac{m_e c^2 \sqrt{1 - \frac{\alpha^2}{n^2}} \sqrt{1 - \frac{\alpha^2}{n^2}}}{\sqrt{1 - \left(\frac{\alpha}{n}\right)}} - m_e c^2$$

(B.38)
Eq. (B.40) is graphed in Figure (B1) and perfectly intersects the rest mass of the electron, 510998.896 eV, at orbit state \( n = \alpha \) where \( \alpha \) is equal to 1/137.035999 and is the fine structure constant.

\[
\beta E = m_e c^2 \sqrt{1 - \left(\frac{\alpha}{n}\right)^2} - m_e c^2 = m_e c^2 \left( \sqrt{1 - \left(\frac{\alpha}{n}\right)^2} - 1 \right)
\]  

(B.39)

Multiplying all terms by -1 gives the binding energy equation

\[
E_B = m_e c^2 \left( 1 - \sqrt{1 - \left(\frac{\alpha}{n}\right)^2} \right)
\]  

(B.40) 
(similar to GUTCP Eq. (1.294))

Eq. (B.40) is graphed in Figure (B1) and perfectly intersects the rest mass of the electron, 510998.896 eV, at orbit state \( n = \alpha \) where \( \alpha \) is equal to 1/137.035999 and is the fine structure constant.

The binding energy at any orbit state is equal to the energy of the photon emitted when a free electron having rest mass \( m_{e0} \) starts at infinity and is captured by a proton and becomes bound at some final orbit state \( n \). Since mass (and therefore energy) is conserved, the final mass of the electron as it orbits the proton is reduced by the mass equivalent of the photon emitted. Designating the mass equivalent of this photon energy as \( m_{\text{photon}} \) and the mass of the orbiting electron as \( m_{\text{orbit}} \), the equation is

\[
m_{\text{photon}} = m_{e0} - m_{\text{orbit}}
\]  

(B.41)

Eq. (B.41) can be rearranged as

\[
m_{\text{orbit}} = m_{e0} - m_{\text{photon}}
\]  

(B.42)

since \( m_{\text{photon}} \) in terms of energy is

\[
m_{\text{photon}} = \frac{E_B}{c^2}
\]  

(B.43)
The difference between the negative of the potential energy and the kinetic energy is equal to the binding energy:

\[-PE - KE = E_B\]

510998.896 eV at \(n = 1/137.035999\)

Figure B1. Hydrogen: Relativistic Potential Energy, Kinetic Energy and Binding Energy.

Figure (B1) is a graph of the following relativistic energies: Relativistic Potential Energy from Eq. (B.30), Relativistic Kinetic Energy from Eq. (B.32) and the Relativistic Binding Energy from Eq. (B.40).
Inserting Eq. (B.43) into Eq. (B.42) gives

$$m_{\text{orbit}} = m_{e0} - \frac{E_B}{c^2} \quad \text{(B.44)}$$

Inserting the binding energy equation (Eq. (B.40)) into Eq. (B.44) gives

$$m_{\text{orbit}} = m_{e0} - \frac{m_e c^2}{c^2} \left(1 - \sqrt{1 - \left(\frac{\alpha}{n}\right)^2}\right) = m_{e0} - m_{e0} \left(1 - \sqrt{1 - \left(\frac{\alpha}{n}\right)^2}\right)$$

Which reduces to

$$m_{\text{orbit}} = m_{e0} - m_{e0} + m_{e0} \sqrt{1 - \left(\frac{\alpha}{n}\right)^2}$$

And results in the equation for the relativistic mass of the electron as it orbits the proton as a function of orbit state $n$:

$$m_{\text{orbit}} = m_{e0} \sqrt{1 - \left(\frac{\alpha}{n}\right)^2} \quad \text{(B.45)}$$

Eq. (B.45) can be interpreted as the mass of the electron decreases as the orbit state drops to lower fractional orbit states through the emission of a photon. If the electron could get to $n = \alpha$ where $r = \alpha a_0$ and is the particle production radius, then its mass would be zero. But, see pages 84 and 85 of this document where it hypothesizes the electron can not fall lower than $n = 1/84$ through the release of a photon due to its orbital velocity reaching the speed of light.
The following is speculation and I am not sure if it is correct:

Eq. (B.23) is the relativistic radius, \( r_n \), for the orbiting electron and Eq. (B.45) is the relativistic mass, \( m_{\text{orbit}} \), for the orbiting electron. GUTCP states the angular momentum is equal to \( \hbar \) and is relativistically invariant. The angular momentum equation for the orbiting electron is then

\[
m_{\text{orbit}} v r_n = \hbar \tag{B.46}
\]

Inserting relativistic radius equation (Eq. (B.23)) and relativistic mass equation (Eq. (B.45)) into Eq. (B.46) gives

\[
\left( m_e 0 \sqrt{1-\left(\frac{\alpha}{n}\right)^2} \right) v \left( a_0 n \sqrt{1-\left(\frac{\alpha}{n}\right)^2} \right) = \hbar \tag{B.47}
\]

Solving Eq. (B.47) for velocity gives

\[
v_{\text{orbit}} = \frac{\hbar}{m_e a_0 n \left(1-\left(\frac{\alpha}{n}\right)^2\right)} \tag{B.48}
\]

Eq. (B.48) is the velocity of the electron as it orbits the proton as a function of orbit state \( n \) and is designated \( v_{\text{orbit}} \) and is termed the “relativistic velocity”.

With this new notation, Eq. (B.46) can be rewritten as

\[
m_{\text{orbit}} v_{\text{orbit}} r_n = \hbar \tag{B.49}
\]

Figure (B2) is a graph of the following parameters for the electron as it orbits the proton:

- \( m_{\text{orbit}} \) from Eq. (B.45)
- \( v_{\text{orbit}} \) from Eq. (B.48)
- \( r_n \) from Eq. (B.23)
Figure (B2) is a graph of the relativistic parameters $m_{\text{orbit}}$ (orbiting electron mass), $v_{\text{orbit}}$ (orbiting electron velocity) and $r_n$ (orbiting electron radius) for the electron in the hydrogen atom. The angular momentum is relativistically invariant and equal to $\hbar$ and the following equation for angular momentum applies at all orbit states:

$$L = m_{\text{orbit}} v_{\text{orbit}} r_n = \hbar$$

The next page discusses the possibility that the bound electron cannot fall lower than orbit state $n = 1/84$ through the emission of a photon due to the fact that the electron would orbit faster than the speed of light at any lower orbit state.
The following is not in GUTCP and may be incorrect and should be considered as a hypothesis.

Near $n = 1/84$ in Figure (B2), the orbiting electron has a mass of $7.20 \times 10^{-31}$ kg which is about 21% less mass than its original rest mass $m_{e0} = 9.109 \times 10^{-31}$ kg and is orbiting near the speed of light. It appears that an electron cannot drop lower than $n = 1/84$ through emission of a photon since dropping to $n = 1/85$ would require it to have a velocity faster than the speed of light. The following hypothesis may (or may not) be correct:

1. Figure (B2) is correct and represents the situation where an electron emits a photon through conversion of its own potential energy to photon energy.
2. Figure (1) on page 6 is also correct but Figure (1) represents the situation where the electron is physically compressed with an outside energy source. Figuratively speaking, this could be done using an infinite number of movable mirrors that confine it to a smaller and smaller spherical volume. In this scenario, there is no proton at the center and no photons are emitted at any orbit states during the compression. All of the energy put into the compression/moving mirrors stays with the electron as it gets smaller and smaller until finally it reaches its particle production radius, $r = \alpha a_0$, where the electron orbits at light speed $c$, becomes a photon and has 510998.896 eV of energy. Energy is conserved in this scenario since the external energy that was put into the compression of the mirrors could theoretically come from converting one of the two 510998.896 eV photons emitted from the original electron/positron particle production into mechanical energy that moves the mirrors. This scenario of mirrors compressing the electron into a smaller volume could be viewed as blue shifting a free electron to higher and higher frequencies until its frequency matches that of a photon having 510998.896 eV of energy.

The creation of a free electron and positron from a 1.02 MeV photon during particle production can be viewed in thermodynamic terms as the electron undergoing free expansion, similar to a gas undergoing free expansion. The hypothetical compression of the electron using an infinite number of infinitesimal movable mirrors to a smaller and smaller size until it eventually becomes a photon at a radius equal to the particle production radius could be viewed in thermodynamic terms as adiabatic compression. During particle production, the photon must first collide with a stationary particle to absorb linear momentum.
An electron starting at infinite distance from a proton with zero velocity starts with a rest mass equal to $m_{e0}$ and releases a photon when it becomes bound to a proton (i.e. orbits a proton). The resulting mass of the orbiting electron $m_{\text{orbit}}$ is smaller than $m_{e0}$ by an amount that exactly matches the mass equivalent of the photon it just emitted. The mass of the orbiting electron $m_{\text{orbit}}$ is only a function of the orbit state $n$. The smallest orbit that an electron can orbit the proton by “falling” from a higher orbit and releasing a photon is $n = 1/84$ since releasing a photon and falling to $n = 1/85$ would result in an orbit velocity faster than the speed of light. The mass $m_{\text{orbit}}$ of the electron at $n = 1/84$ is $7.20 \times 10^{-31}$ kg (21% less mass than a free electron having $9.109 \times 10^{-31}$ kg) which corresponds to a rest mass of 403.7 keV and has infinitesimal masses and charges orbiting at about $0.98c$. Theoretically, if it “fell” any closer to the proton, it could gain enough energy (approximately 0.5 keV) and become a photon with approximately 403 keV that travels off in a straight line at the speed of light $c$ or possibly it could orbit the proton at the speed of light $c$. 
Appendix C

Relativistic Equations For Electron In Hydrogen With The Reduced Mass Factor.
The derivation of the binding energy $E_B$ for excited and fractional orbits of hydrogen shown in this appendix includes the reduced mass factor (note: the binding energy for the hydrogen atom is the same as ionization energy). The photon energy released by hydrogen for a transition between two orbit states can be calculated through subtraction of the binding energy for each of the two orbit states. GUTCP discusses the equations for the binding energy (and the principal energy levels) near GUTCP Eqs. (I.1), (I.45), (I.97), (I.118), (I.124) and 1.290.

The derivation in this appendix is based on a model of two charged point masses, the electron and the proton, where the electron orbits in a circle around the proton about their combined center of mass (the barycenter). This derivation assumes the proton is not fixed in space and thus it includes the reduced mass effect. In this derivation, assuming the proton is not fixed in space increases the complexity of the derivation and results in a more complex equation for binding energy $E_B$ as seen in the resulting equation (Eq. (C.52)).

Mills writes in GUTCP that the reduced mass effect is due to a magnetic interaction between the orbiting electron and the proton. Mills terms this force the $F_{mag}$ force and discusses it near GUTCP Eq. (I.92), (1.246) and (1.253).

The derived equation for the binding energy of hydrogen without the reduced mass factor (in Appendix B) results in a calculated value of 13.606 eV for the ground orbit state. The derived equation for the binding energy of hydrogen with the reduced mass factor included (in this appendix) results in a calculated value of 13.598 eV for the ground orbit state $n = 1$ which virtually, but not exactly, matches the measured ionization energy of 13.598 eV (using more decimal places would show a difference). The reduced mass factor is approximately equal to $1836.15/(1836.15 + 1) = .9994557$ where 1836.15 is the ratio of the mass of the proton to the mass of the electron. Multiplying 13.606 eV by .9994557 (the reduced mass factor) results in 13.598 eV. The point mass model with the two masses orbiting about their common center of mass can be seen in the equations for the “reduced mass” concept in standard atomic physics (particularly spectroscopy data) and even in binary stars where the two stars “wobble” as viewed from earth due to the two masses orbiting about their common center of mass.
The model is based on a negatively charged point mass $m_e$ with charge $-e$ (the electron) and a positively charged point mass $m_p$ with charge $+e$ (the proton) orbiting around their common center of mass (barycenter) at combined radius $r$. The distance from center of mass to the point sized electron is $r_1$ and the distance from the center of mass to the point sized proton is $r_2$.

Note: This model shown at left is just for setting up the math equations which match the spectroscopy data. See page 20 and 21 of this presentation that shows GUTCP’s spherical shell model.

- **Proton**
  - $m_p = 1.67262 \times 10^{-27}$ kg
  - $+e = +1.60217 \times 10^{-19}$ C
  - $r_2 = $ radius to orbit center point

- **Electron**
  - $m_e = $ orbiting mass varies as a function of orbit state $n$
  - $-e = -1.60217 \times 10^{-19}$ C
  - $r_1 = $ radius to orbit center point

combined distance between electron and proton:
\[ r = r_1 + r_2 \]
Formula for center of mass for scenario depicted on previous page

\[ m_e r_1 = m_p r_2 \quad (C.1) \]

Solving Eq. (C.1) for \( r_1 \) and \( r_2 \) gives

\[ r_1 = \frac{m_p r_2}{m_e} \quad (C.2) \]

\[ r_2 = \frac{m_e r_1}{m_p} \quad (C.3) \]

The combined radius between the electron and the proton is

\[ r = r_1 + r_2 = \frac{m_p r_2}{m_e} + r_2 = r_2 \left( \frac{m_p}{m_e} + 1 \right) \quad (C.4) \]

Solving Eq. (C.4) for \( r_2 \) gives

\[ r_2 = \frac{r}{\left( \frac{m_p}{m_e} + 1 \right)} = \frac{r}{\left( \frac{m_p + m_e}{m_e} \right)} = \frac{m_e r}{(m_p + m_e)} \quad (C.5) \]
Inserting Eq. (C.5) into Eq. (C.2) gives an equation for $r_1$

$$r_1 = \frac{m_pr_2}{m_e} = \frac{m_e r}{(m_e + m_p)} = \frac{m_pr}{(m_e + m_p)} \quad (C.6)$$

The angular momentum $L$ of the proton/electron system is equal to $\hbar$ at all orbit states. The angular momentum is equal to the moment of inertia $I$ multiplied by the angular velocity $\omega$:

$$L = I\omega = \hbar \quad (C.7)$$

The moment of inertia for each point particle orbiting around a center point is

$$I = mr^2$$

The moment of inertia for the proton/electron system is the sum of the individual moments of inertia:

$$I = m_e r_1^2 + m_p r_2^2 \quad (C.8)$$

Inserting the formulas for $r_1$ (Eq.(C.6)) and $r_2$ (Eq. (C.5)) into Eq. (C.8) gives

$$I = m_e \left( \frac{m_pr}{(m_e + m_p)} \right)^2 + m_p \left( \frac{m_er}{(m_e + m_p)} \right)^2 \quad (C.9)$$
Expanding and simplifying Eq. (C.9):

\[ I = \frac{m_e m_p r^2}{(m_e + m_p)^2} + \frac{m_p m_e r^2}{(m_e + m_p)^2} = \frac{m_p m_e r^2 (m_e + m_p)}{(m_e + m_p)^2} \]

\[ I = \frac{m_p m_e r^2}{(m_e + m_p)} \]  

(C.10)

Inserting Eq. (C.10) into the angular momentum equation (Eq. (C.7)) gives

\[ L = I \omega = \left( \frac{r^2 m_p m_e}{(m_e + m_p)} \right) \omega = \hbar \]  

(C.11)

Angular velocity \( \omega \) equals the orbit velocity \( v \) divided by the radius \( r \)

\[ \omega = \frac{v}{r} \]  

(C.12)

Inserting angular velocity equation (Eq. (C.12)) into angular momentum equation (Eq. (C.11)) gives

\[ L = \left( \frac{r^2 m_p m_e}{(m_e + m_p)} \right) v = \hbar \]  

(C.13)
Solving Eq. (C.13) for \( r \) gives

\[
r = \frac{\hbar (m_e + m_p)}{m_p m_e v}
\]  \hspace{1cm} (C.14)

The radius equation above contains the orbit velocity \( v \) which is governed by the force balance equation between the proton and the orbiting electron where the centripetal acceleration force equals the electrostatic force:

\[
\frac{m_e v^2}{r} = \frac{e^2}{4 \pi \epsilon_0 r^2}
\]  \hspace{1cm} (C.15)

But in GUTCP, there is the postulate that the charge that the electron experiences due to the proton is affected by orbit state \( n \) due to the trapped photon and this results in \( e^2 \) in the equation being replaced by \( \frac{e^2}{n} \). The first postulate is introduced here in the force balance equation where the electrostatic force equation has \( e^2 \) replaced by \( \frac{e^2}{n} \) due to the trapped photon.

So Eq. (C.15) becomes

\[
\frac{m_e v^2}{r} = \frac{e^2}{4 \pi \epsilon_0 n r^2}
\]  \hspace{1cm} (C.16)

Solving Eq. (C.16) for velocity \( v \) gives

\[
v = \sqrt{\frac{e^2}{4 \pi \epsilon_0 m_e n r}}
\]  \hspace{1cm} (C.17)
Inserting velocity equation (Eq. (C.17)) into the radius equation (Eq. (C.14)) gives

\[ r = \frac{\hbar (m_e + m_P)}{m_p m_e \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e n r}}} = \frac{\hbar}{\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e n r}}} \left( \frac{m_e + m_P}{m_p m_e} \right) \]

Squaring both sides gives

\[ r^2 = \frac{\hbar^2 4\pi\varepsilon_0 m_e n r}{e^2} \left( \frac{m_e + m_P}{m_p m_e} \right)^2 \]  \hspace{1cm} (C.18)

Solving Eq. (C.18) for \( r \) gives

\[ r = \frac{\hbar^2 4\pi\varepsilon_0 m_e n}{e^2} \left( \frac{m_e + m_P}{m_p m_e} \right)^2 \]  \hspace{1cm} (C.19)

The bohr radius equals

\[ a_0 = \frac{\hbar^2 4\pi\varepsilon_0}{m_e_0 e^2} \]  \hspace{1cm} (C.20)

Moving \( m_{e_0} \) in Eq. (C.20) from the right side to the left side gives:

\[ a_0 m_{e_0} = \frac{\hbar^2 4\pi\varepsilon_0}{e^2} \]  \hspace{1cm} (C.21)
Inserting Eq. (C.21) into Eq. (C.19) gives

\[ r = a_0 m_{e_0} m_e n \left( \frac{m_e + m_p}{m_p m_e} \right)^2 \]  \hspace{1cm} (C.22)

The term in brackets can be rewritten as

\[ \frac{m_e + m_p}{m_p m_e} = \frac{1}{m_e} + \frac{1}{m_p} \]  \hspace{1cm} (C.23)

Inserting Eq. (C.23) into Eq. (C.22) gives

\[ r = n a_0 m_{e_0} m_e \left( \frac{1}{m_e} + \frac{1}{m_p} \right)^2 = n a_0 m_{e_0} m_e \left( \frac{1}{m_e^2} + \frac{2}{m_e m_p} + \frac{1}{m_p^2} \right) \]  \hspace{1cm} (C.24)

Pulling \( m_e^2 \) out of the term in brackets in Eq. (C.24) gives

\[ r = \frac{n a_0 m_{e_0} m_e}{m_e^2} \left( \frac{m_e^2}{m_e^2} + \frac{2m_e^2}{m_e m_p} + \frac{m_e^2}{m_p^2} \right) = \frac{n a_0 m_{e_0}}{m_e} \left( 1 + \frac{2m_e}{m_p} + \frac{m_e}{m_p^2} \right) \]  \hspace{1cm} (C.25)

(similar to GUTCP Eq. (1.282))
Eq. (C.25) shown above is for the radius of the hydrogen atom and is the same as GUTCP Eq. (1.282) except mass number $A$ and charge number $Z$ are not included and there is a “2” in the numerator of the middle term in brackets. Eq. (C.25) also includes orbit state $n$ and a third term (with blue arrow pointing at it) in the brackets which is not in GUTCP Eq. (1.282) but this third term has the mass of the proton squared in the denominator and is insignificant at all orbit states since it contributes a maximum of 1 part in 3.4 million to the overall result (i.e. $(1/1836)^2$).

Eq. (C.25) can be rearranged and written as the following which will be used further down in the derivation:

$$r = n a_0 m_{e0} \left( 1 + \frac{2m_e}{m_p} + \frac{m_e^2}{m_p^2} \right)$$

(C.26)

$\left(\text{Similar to GUTCP Eq. (1.282)}\right)$

$m_{e0}$ in Eq. (C.25) and Eq. (C.26) is the rest mass for the electron and is a constant and equal to $9.109382 \times 10^{-31}$ kg but $m_e$ is the relativistic mass of the electron and varies as a function of velocity as given by the Lorentz transformation:

$$m_e (v) = m_e = \frac{m_{e0}}{\sqrt{1-\beta^2}} = \frac{m_{e0}}{\sqrt{1-(v/c)^2}}$$

(C.27)

(same as GUTCP Eq. (1.286))
An equation for the velocity of the electron in the hydrogen atom can be obtained from the condition that the electron/proton system orbits with an angular momentum of $\hbar$ (the reduced Planck constant) at all orbit states $n$.

Angular momentum: $L = m_e v r = \hbar$  

Solving Eq. (C.28) for velocity gives

$$v = \frac{\hbar}{m_e r}$$

(C.28)

(similar to GUTCP (1.61 and 1.35))

Dividing both sides of Eq. (C.29) by $c$ gives the relativistic parameter $\beta$:

$$\beta = \frac{v}{c} = \frac{\hbar}{m_e c r}$$

(C.29)

(same as GUTCP Eq. (1.283))

Inserting the radius equation (Eq. (C.25)) into the relativistic parameter equation (Eq. (C.30)) gives

$$\beta = \frac{v}{c} = \frac{\hbar}{m_e c n a_0 m_e 0 \left(1 + \frac{2m_e}{m_p} + \frac{m_e^2}{m_p^2}\right)} = \frac{\hbar}{c n a_0 m_e 0 \left(1 + \frac{2m_e}{m_p} + \frac{m_e^2}{m_p^2}\right)}$$

(C.31)

(similar to GUTCP Eq. (1.283))
The equation for the fine structure constant $\alpha$ is

$$\alpha = \frac{\hbar}{m_e c a_0}$$  \hspace{1cm} (C.32)

(same as GUTCP Eq. (1.284))

Inserting the fine structure constant equation (Eq. (C.32)) into Eq. (C.31) gives

$$\beta = \frac{v}{c} = \frac{\hbar}{cn a_0 m_e (1 + \frac{2m_e}{m_p} + \frac{m_e^2}{m_p^2})} = \frac{\alpha}{n (1 + \frac{2m_e}{m_p} + \frac{m_e^2}{m_p^2})}$$  \hspace{1cm} (C.33)

(similar to GUTCP Eq. (1.285))

At this point, GUTCP states that the velocity equation above needs a correction due to an interaction between the invariant magnetic moments of the proton and the electron and inserts a “2” in the middle term in brackets in the denominator of Eq. (C.33). The reader can go to GUTCP Eqs. (1.287) and (1.288) to see the full explanation. The result is that Eq. (C.33) becomes

$$\beta = \frac{v}{c} = \frac{\alpha}{n (1 + \frac{2m_e}{2m_p} + \frac{m_e^2}{m_p^2})}$$  \hspace{1cm} (C.34)

(similar to GUTCP Eq. (1.288))

Note: this presentation will leave the “2” visible in the denominator of the middle term in brackets (with blue arrow pointing at it) for the rest of this appendix and not cancel it out with the “2” in the numerator.
And the electron relativistic mass equation (Eq. (C.27)) becomes

\[
m_e = \frac{m_{e0}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{m_{e0}}{\sqrt{1-\alpha^2}}
\]

\[
\left[1+\frac{2m_e}{2m_p} + \frac{m_e^2}{m_p^2}\right]
\]

(C.35)

The right side of the equation above is the relativistic mass for the orbiting bound electron. To make things clearer, this presentation will use the notation \(m_{\text{orbit}}\) for the relativistic mass for the orbiting bound electron (which is a function of \(n\)) to distinguish it from \(m_e\) which is the relativistic mass of a free space electron with straight line velocity \(v\). The result of this notation change results in Eq. (C.35) becoming:

\[
m_{\text{orbit}} = \frac{m_{e0}}{\sqrt{1-\alpha^2}}
\]

\[
\left[1+\frac{2m_{\text{orbit}}}{2m_p} + \frac{m_{\text{orbit}}^2}{m_p^2}\right]
\]

(C.36)

It can be noted that \(m_{\text{orbit}}\) is on both the right and left side in the equation above. This derivation solves this issue by iteratively calculating \(m_{\text{orbit}}\) for use in the energy equations.
Inserting the notation change of $m_e$ to $m_{\text{orbit}}$ into Eq. (C.26) gives

$$r = na_0 \left( \frac{m_{e0}}{m_{\text{orbit}}} + \frac{2m_{e0}}{m_P} + \frac{m_{e0} m_{\text{orbit}}}{m_P^2} \right)$$

(C.37)

Inserting Eq. (C.36) into the first term (but not the third term) of the term in brackets in Eq. (C.37) and then simplifying gives

$$r = na_0 \left( \sqrt{1 - \left( \frac{\alpha}{n \left( 1 + \frac{2m_{\text{orbit}}}{2m_P} + \frac{m_{\text{orbit}}^2}{m_P^2} \right)} \right)^2} + \frac{2m_{e0}}{m_P} + \frac{m_{e0} m_{\text{orbit}}}{m_P^2} \right)$$

(C.38)

(similar to GUTCP Eq. (1.289))

$m_{\text{orbit}}$ was inserted into the first term in brackets in Eq. (C.37) but not into the third term to keep Eq. (C.38) simpler and due to the fact that the third term in each of the brackets are divided by $(m_P)^2$ and contribute a maximum of only 1 part in 3.4 million to the overall result (i.e. $1/1836^2$). Going forward, these two terms (with blue arrows pointing at them) will be deleted for simplification reasons. Eq. (C.38) then becomes:

$$r_n = na_0 \left( \sqrt{1 - \left( \frac{\alpha}{n \left( 1 + \frac{2m_{\text{orbit}}}{2m_P} \right)} \right)^2} + \frac{2m_{e0}}{m_P} \right)$$

(C.39)

(similar to GUTCP Eq. (1.289))
Eq. (C.39) shown on the previous page is the radius of the hydrogen atom as a function of orbit state \( n \) and is the same as GUTCP Eq. (1.289) except that it includes orbit state \( n \) and there is a “2” in the numerator of the two terms that have \( m_P \) in the denominator. Also, as was noted previously, it does not include nuclear charge number \( Z \) and atomic mass number \( A \) since both are 1 for the hydrogen atom.

The total energy of the electron \( E_{\text{Total}} \) is the sum of the potential energy \( V_n \) and the kinetic energy \( T_n \) which are each functions of orbit state \( n \).

\[
E_{\text{Total}} = V_n + T_n
\]  
(C.40)  
(same as GUTCP Eq. (1.290))

The potential energy equation for the electron is

\[
V = -\frac{e^2}{(4\pi\varepsilon_0)r}
\]  
(C.41)  
(similar to GUTCP Eq. (1.261) except inserting \( r_n \) for \( r_1 \) and setting \( Z = 1 \))

But in GUTCP, there is the postulate that the charge that the electron experiences due to the proton is affected by orbit state \( n \) due to the trapped photon and this results in \( e^2 \) in the equation being replaced by \( \frac{e^2}{n} \). Thus Eq. (C.41) becomes

\[
V_n = -\frac{e^2}{(4\pi\varepsilon_0)n r_n}
\]  
(C.42)  
The second postulate is introduced here in the potential energy equation where the electrostatic potential energy equation has \( e^2 \) replaced by \( e^2/n \) due to the trapped photon.
Inserting the radius equation (Eq. (C.39)) into the potential energy equation (Eq. (C.42)) gives

\[
V_n = -\frac{e^2}{(4\pi\varepsilon_0)n^2a_0\sqrt{1-\left(\frac{\alpha}{n\left(1 + \frac{2m_{\text{orbit}}}{2m_p}\right)}\right)^2} + \frac{2m_{e0}}{m_p}}
\]  \hspace{1cm} (C.43)

The bohr radius is

\[
a_0 = \frac{\hbar^2 4\pi\varepsilon_0}{m_{e0}e^2}
\]  \hspace{1cm} (C.44)

The fine structure constant is

\[
\alpha = \frac{\hbar}{m_{e0}ca_0}
\]  \hspace{1cm} (C.45)

Inserting the bohr radius equation (Eq. (C.44)) and the fine structure constant equation (Eq. (C.45)) into Eq. (C.43) and simplifying results in

\[
V_n = -\frac{m_{e0}c^2\alpha^2}{n^2\sqrt{1-\left(\frac{\alpha}{n\left(1 + \frac{2m_{\text{orbit}}}{2m_p}\right)}\right)^2} + \frac{2m_{e0}}{m_p}}
\]  \hspace{1cm} (C.46)
The kinetic energy for the electron when it is traveling in a straight line with velocity \( \mathbf{v} \) is

\[
T = m_{e0} c^2 \left( \frac{1}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} - 1 \right)
\]

(C.47)

(same as GUTCP Eq. (1.291))

Inserting the relativistic parameter \( \beta \) (Eq. (C.34)) into Eq. (C.47) and neglecting the term with \( (m_p)^2 \) in the denominator (because it is too small) gives the relativistic kinetic energy for the orbiting electron:

\[
T_n = m_{e0} c^2 \left( \frac{1}{\sqrt{1 - \left( \frac{\alpha}{n(1 + \frac{2m_{\text{orbit}}}{2m_p})} \right)^2}} - 1 \right)
\]

(C.48)
Inserting the potential energy (Eq. (C.46)) and kinetic energy (Eq. (C.48)) equations into the total energy equation gives

\[
E_{\text{Total},n} = -\frac{m_e c^2 \alpha^2}{n^2 \left(1 - \frac{\alpha}{n \left(1 + \frac{2m_{\text{orbit}}}{2m_p}\right)}\right)^2} + m_e c^2 \left(1 - \frac{\alpha}{n \left(1 + \frac{2m_{\text{orbit}}}{2m_p}\right)}\right)^2 - 1
\]

Total energy, \(E_{\text{Total},n}\) = potential energy, \(V\) + kinetic energy, \(T\)

Next, the binding energy \(E_B\) is the energy required to ionize the electron (i.e. remove the bound electron from the atom out to a distance of infinity). The binding energy is the difference in total energy of the electron between final orbit state \(n_f = \infty\) and initial orbit state \(n_i\).

\[
E_B = \Delta E_{\text{Total}} = E_{\text{Total},f} - E_{\text{Total},i}
\]

Where the subscript “\(f\)” is for the final orbit state and “\(i\)” is for the initial orbit state. At orbit state \(n = \infty\), the total energy, \(E_{\text{total},f}\), is equal to zero and the binding energy (Eq. (C.52)) becomes:

\[
E_B = \Delta E_{\text{Total}} = 0 - E_{\text{Total},i} = -E_{\text{Total},i}
\]

Thus for a given orbit state \(n\), the binding energy \(E_B\) is the negative of the total energy.
Inserting Eq. (C.49) into Eq. (C.51) gives an equation for the binding energy:

\[
E_B = \frac{m_e \alpha^2}{n^2} \left[ \left( \frac{\alpha}{n \left( 1 + \frac{2m_{\text{orbit}}}{2m_p} \right)} \right)^2 + \frac{2m_e}{m_p} \right] - m_e c^2 \left[ \left( \frac{1}{n \left( 1 + \frac{2m_{\text{orbit}}}{2m_p} \right)} \right)^2 - 1 \right] \tag{C.52}
\]

An equation for \( m_{\text{orbit}} \) for use in Eq. (C.52) is given further below in Eq. (C.57).

Eq. (C.52) is important because it shows the full relativistic energy equation where the binding energy is equal to the kinetic energy subtracted from the negative of the potential energy.*

*The potential energy is either zero or a negative number as a result of the standard convention of defining the potential energy as zero at infinite distance. Therefore the *negative* of the potential energy is always a positive number or zero. And the kinetic energy is always a positive number or zero.
If the spin-nuclear effect is assumed to be negligible (i.e. little or no reduced mass effect) then Eq. (C.52) reduces to Eq. (B.40) from Appendix B:

\[
E_B = m_e c^2 \left( 1 - \sqrt{1 - \left( \frac{\alpha}{n} \right)^2} \right)
\]

(A similar to GUTCP Eq. (1.294))

A derivation of Eq. (B.40) can be seen in Appendix B.

Graphs for excited and fractional orbit states of hydrogen using Eq. (B.40) are shown in Figures (B1) and (B2).

The binding energy is equal to the energy of the photon emitted when an electron having rest mass \( m_{e0} \) starts at infinity and is captured by a proton and becomes bound at some final orbit state \( n \). The final mass of the electron as it orbits the proton is reduced by the mass equivalent of the photon emitted. Designating the mass equivalent of this photon energy as \( m_{\text{photon}} \) and the mass of the orbiting electron as \( m_{\text{orbit}} \), the equation is

\[
m_{\text{photon}} = m_{e0} - m_{\text{orbit}}
\]

(Eq. (C.53))

Eq. (C.53) can rearranged as

\[
m_{\text{orbit}} = m_{e0} - m_{\text{photon}}
\]

(Eq. (C.54))

since \( m_{\text{photon}} \) in terms of energy is

\[
m_{\text{photon}} = \frac{E_B}{c^2}
\]

(Eq. (C.55))

Inserting Eq. (C.55) into Eq. (C.54) gives

\[
m_{\text{orbit}} = m_{e0} - \frac{E_B}{c^2}
\]

(Eq. (C.56))
Inserting Eq (C.52) into and Eq. (C.56) gives

\[
m_{\text{orbit}} = m_{e0} - \frac{1}{c^2} \left( \frac{m_{e0} c^2 \alpha^2}{n^2 \left( 1 - \frac{\alpha}{n \left( 1 + \frac{2m_{\text{orbit}}}{2m_P} \right) } \right)^2} + \frac{2m_{e0}}{m_P} \right)
- m_{e0} c^2 \left( \frac{1}{n \left( 1 + \frac{2m_{\text{orbit}}}{2m_P} \right) } \right)^2 - 1 \]  

(C.57)
Eq. (C.52) for the binding energy requires that Eq. (C.57) for \( m_{\text{orbit}} \) be solved first. An iterative process can be done:

**Iterative process for calculating binding energy \( E_B \) for a given orbit state \( n \):**

**Step 1.**
First iteration: Calculate \( m_{\text{orbit}} \) from Eq. (C.57). Assume \( m_{\text{orbit}} \) on the right side of the equation is zero on this first iteration.

**Step 2.**
Second iteration: Use the \( m_{\text{orbit}} \) result from Step 1 and calculate a new \( m_{\text{orbit}} \). This second iteration is usually all that is needed and any further iterations do not significantly change \( m_{\text{orbit}} \). This second iteration slightly affects the mass calculation for the orbiting electron at orbit states that are “more fractional” (i.e. orbit states with smaller radii) than \( n = 1/100 \) and a 3\(^{rd}\) iteration does virtually nothing.

**Step 3.**
Calculate the binding energy \( E_B \) from Eq. (C.52) using \( m_{\text{orbit}} \) from Step 2.

Problems arise using Microsoft Excel for binding energy calculations using Eq. (C.52) and the software seems to have difficulties as \( n \) approaches \( n = 1/137 \) due to two large numbers being subtracted from each other. For example, at \( n = 1/137 \), Eq. (C.52) is subtracting 20 million from 20.5 million and various errors are more visible (possibly errors in the constants or round off errors inside the computer itself etc.) with the end result that Eq. (B.40) seems to be much more accurate for orbit states near \( n = 1/137 \). As a result, the graphs that use Eq. (C.52) (specifically Figure (A3)) in this presentation use Eq. (B.40) for only the data points \( n = 1/137 \) and \( n = \alpha = 1/137.035999 \).

There are potentially methods to minimizing this calculation problem, one is a more powerful computer or software. Another is to rewrite the equation to minimize the subtraction of large numbers (but not sure if this is possible). But these solutions will only be needed for orbit states near \( n = 1/137 \) since any calculation at \( n = \alpha = 1/137.035999 \) must be done using Eq. (B.40) so that infinities are not created.
In summary, an electron at infinite distance from a proton starts with a rest mass equal to \( m_{e0} \) and releases a photon when it becomes bound to a proton (i.e. orbits a proton). The resulting mass of the orbiting electron, \( m_{\text{orbit}} \), is smaller than \( m_{e0} \) by an amount that exactly matches the mass equivalent of the photon it just released. The mass of the orbiting electron, \( m_{\text{orbit}} \), is only a function of the orbit state \( n \). The mass of the orbiting electron, \( m_{\text{orbit}} \), decreases as the orbit state decreases. The mass of the orbiting electron plus the mass of the photons it released to get to that orbit state (starting when it was a free electron) plus the mass equivalent (from \( E=mc^2 \)) of any resonant energy transfer process equals the rest mass of the free electron \( m_{e0} \) or:

\[
m_{e0} = m_{\text{orbit}} + m_{\text{resonant}} + m_{\text{photon}}
\]  

(C.57)

Where

\[
m_{e0} = 9.109382 \times 10^{-31} \text{ kg} = \text{rest mass of electron}
\]

\[
m_{\text{orbit}} = \text{mass of orbiting electron in hydrogen atom}
\]

\[
m_{\text{resonant}} = \text{mass equivalent of Forster Resonant Energy Transfer process (FRET)}
\]

\[
m_{\text{photon}} = \text{mass equivalent of all emitted photons}
\]
Appendix D

Why Do Relativistic Equations Match Non-Relativistic Equations near n = 1?
Mills’s book *The Grand Unified Theory of Classical Physics* (GUTCP) makes the following statement in the Introduction: “Consider that in fact, the mathematics of the three theories of Bohr, Schrödinger, and presently CP converge to Eq. (I.1) as the principal energy levels of the hydrogen atom.” [Note: CP stands for Classical Physics which GUTCP is based on.]

\[
E_n = -\frac{e^2}{n^2 8\pi\varepsilon_0 a_H} = \frac{-13.598 ~\text{eV}}{n^2} \quad \text{Eq. (D.1)}
\]

(Similar to GUTCP Eqs. I.1, I.45, I.97, I.118, I.124 and 1.290)

GUTCP derives Eq. (D.1) (i.e. GUTCP Eq. (I.1)) using the following equation for kinetic energy, \( T \)

\[
T = \frac{1}{2}mv^2 \quad \text{(Note: “T” represents kinetic energy)} \quad \text{Eq. (D.2)}
\]

This works well for orbit states near \( n = 1 \) where the electron orbit velocity is much less than the speed of light and \( v \ll c \) (such as \( n = 3 \) or \( n = 1 \) or \( n = 1/2 \)). But for the lower fractional orbit states the relativistic kinetic equation is needed for accuracy at the high orbit velocities. The relativistic kinetic energy equation is:

\[
T = m_e c^2 \left( \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) \quad \text{Eq. (D.3)}
\]

(same as GUTCP Eq. (1.291))
The reason that kinetic energy Eq. (D.2) gives accurate answers near $n = 1$ is that it is derived from the relativistic kinetic energy equation from a Taylor series expansion with higher power terms deleted since those higher power terms affect the result insignificantly:

Repeating the relativistic kinetic energy Eq. (D.3):

$$T = m_e c^2 \left( \sqrt{1 - \left( \frac{v}{c} \right)^2} - 1 \right)$$

Eq. (D.3)

(GUTCP Eq. (1.291))

A Taylor series expansion gives the following if terms higher than the 6th power are deleted (i.e. ignored):

$$T = m_e c^2 \left( 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \frac{v^4}{c^4} + \frac{5}{16} \frac{v^6}{c^6} - 1 \right)$$

Eq. (D.4)

At low electron orbit velocities (which occurs near orbit state $n = 1$), terms having velocity $v^4$ divided by speed of light $c^4$ and higher powers can be ignored because they are insignificant. The result is the well known, non-relativistic, classical equation for kinetic energy:

$$T = m_e c^2 \left( 1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) = m_e c^2 \left( \frac{v^2}{2 c^2} \right) = \frac{1}{2} m_e v^2$$

Eq. (D.5)

Classical kinetic energy equation for $v \ll c$
At $n = 1$, the orbit velocity for the electron is $0.007297c$ and the difference between the relativistic binding energy equation from Eq. (B.40) and the non relativistic energy equation from page 46 of this document is $0.0013\%$ which is a small difference (where the value from the non-relativistic energy equation is smaller). But the difference grows as the orbit radius becomes smaller and the electron orbits faster. For example at $n = 1/15$, the electron is orbiting at $0.11c$ and the difference between the relativistic binding energy equation from Eq. (B.40) and the non relativistic binding energy equation from page 46 of this document is $0.3\%$ (where the value from the non-relativistic energy equation is smaller).

At excited orbit states such as $n = 2$ or $n = 3$, the electron is orbiting slowly ($0.0036c$ and $0.0024c$ respectively) and the relativistic binding energy equation gives a value that is very close to the non-relativistic binding energy equation.
Appendix E

Schrodinger Equation vs. GUTCP comparison.
The Introduction of Mills’s book *The Grand Unified Theory of Classical Physics* (GUTCP) has a section titled: “MATHEMATICAL RELATIONSHIP BETWEEN THE THEORIES OF BOHR AND SCHRODINGER WITH RESPECT TO CLASSICAL ATOMIC THEORY“ where the similarities of GUTCP hydrogen electron energy equations are compared with the Schrödinger equation (I do not know if it is referring to the relativistic Schrödinger Equation or the non-relativistic Schrödinger Equation). The chapter also discusses the many failings and inconsistencies of the Schrödinger Equation, specifically that it includes many postulates to obtain measured values and is not based fundamental laws and principles. On page 15 of GUTCP, there is a section titled SHORTCOMINGS OF QUANTUM THEORY AND REASONS FOR A COMPLETE REVISION OF ATOMIC THEORY. Below is a sample of statements from that section:

The Schrödinger equation mathematically gives the Rydberg equation as a set eigenvalues. On this basis alone, it is justified despite its inconsistency with physical laws and numerous experimental observations such as:

- The Schrödinger equation is not Lorentz invariant.
- The Schrödinger equation violates first principles, including special relativity and Maxwell’s equations.
- The Schrödinger equation gives no basis why excited states are radiative and the 13.6 eV state is stable. Mathematics does not determine physics; it only models physics.
- The Schrödinger equation solutions, Eq. (36) and Eq. (37) of Ref. [13], predict that the ground state electron has zero angular energy and zero angular momentum, respectively.
- The Schrödinger equation solution, Eq. (37) of Ref. [13], predicts that the ionized electron may have infinite angular momentum.
- The Schrödinger equation predicts that each of the functions that corresponds to a highly excited state electron is not integrable and cannot be normalized; thus, each is infinite.
- The Schrödinger equation predicts that the ionized electron is sinusoidal over all space and cannot be normalized; thus, it is infinite.
- The Heisenberg Uncertainty Principle arises as the standard deviation in the electron probability wave, but experimentally it is not the basis of wave particle duality.
- The Schrödinger equation does not predict the electron magnetic moment and misses the spin quantum number altogether.
- The Schrödinger equation provides no rational basis for the phenomenon of spin, the Pauli exclusion principle, or Hund’s rules. Instantaneous exchange of information between particles is required, which violates special relativity.
• The Schrödinger equation is not consistent with conservation of energy in an inverse potential field wherein the binding energy is equal to the kinetic energy and the sum of the binding energy and the kinetic energy is equal to the potential energy.

• The Schrödinger equation permits the electron to exist in the nucleus, a state that is physically nonsensical with infinite potential energy and infinite negative kinetic energy.
The similarities between classical physics laws and the Schrodinger Equation can be seen in this snapshot from the website hyperphysics. Notice that the kinetic energy plus the potential energy equals an energy “E”. That energy “E” is the emitted photon energy:
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/schr.html

The Schrödinger equation plays the role of Newton's laws and conservation of energy in classical mechanics - i.e., it predicts the future behavior of a dynamic system. It is a wave equation in terms of the wavefunction which predicts analytically and precisely the probability of events or outcome. The detailed outcome is not strictly determined, but given a large number of events, the Schrödinger equation will predict the distribution of results.

Classical Conservation of Energy
Newton's Laws

\[ \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = E \]
\[ F = ma = -kx \]

Quantum Conservation of Energy
Schrödinger Equation

In making the transition to a wave equation, physical variables take the form of "operators".

\[ \hat{H} = \hat{\Psi} = E \hat{\Psi} \]

Energy "eigenvalue" for the system.

The form of the Hamiltonian operator for a quantum harmonic oscillator.

The kinetic and potential energies are transformed into the Hamiltonian which acts upon the wavefunction to generate the evolution of the wavefunction in time and space. The Schrödinger equation gives the quantized energies of the system and gives the form of the wavefunction so that other properties may be calculated.
The success of GUTCP Eq. (1.293) for the binding energy for one electron atoms and the results of Eq. (C.52) for fractional and excited states of hydrogen show that GUTCP is the simplest and most accurate model for the atom. In GUTCP, the most basic “rules” for the electron are the following:

- The angular momentum of the bound electron is $\hbar$ at all orbit states $n$ (including when it is a free unbound electron).
- The kinetic energy for the electron is based on the relativistic kinetic energy equation (Eq. (B.15)) at all times. This includes when it is bound to a proton (i.e. orbiting the proton) and when it is unbound (i.e. a free electron traveling in space).
- All photons have $\hbar$ (the reduced Planck’s constant) of angular momentum.

The rules above can be seen in equations for Planck’s Radiation Law, the Compton effect, the photoelectric effect, rotational and vibrational Raman spectroscopy and many other “effects” and laws.

There is one postulate in GUTCP for the bound electron that can be hard to understand which is the following:
In GUTCP, there is a trapped photon inside the orbitsphere which results in the electric field strength between the proton and the electron being affected by a factor of $1/n$ for a given orbit states $n$. At the same time, this is one simple postulate can be compared to the multiple confusing postulates of Standard Quantum Mechanics that do not follow classical physics rules.
Appendix F

Non-specific details that didn’t fit into the main section.
Hydrogen at orbit state $n = 1$.

Normal hydrogen in the ground state. Radius is equal to the bohr radius or:
$$ r = a_0 = 0.52946 \text{ Angstroms} $$

Hydrogen at orbit state $n = \alpha = 1/137.035999$. The radius is $137.035999$ times smaller than the normal hydrogen shown at left and its relative size is the size of the period below:

$$ r = \alpha a_0 $$

$$ r = a_0 / 137.035999 = 0.00386 \text{ Angstroms} $$

TSO at orbit state $n = 1/137.035999$
When a hammer strikes a bell, it oscillates at its natural frequency between potential energy (stored stresses in the metal) and kinetic energy (velocity of the vibrating bell surface).

The atomic version of that is when a photon strikes a nucleus and creates a free electron and a free positron during “pair production”.

**Pair Production**

- A volume of space surrounding the nucleus “rings” at its natural frequency between electric energy and magnetic energy.
- 1.022 MeV (or larger) photon strikes nucleus
  - creates a free ionized **positron** having a rest mass of 510998.896 eV
  - creates a free ionized **electron** having a rest mass of 510998.896 eV

The volume of space that “rings” has a radius that is 137.03599 times smaller than normal hydrogen in the ground state (n = 1) and has a frequency that matches a 510998.896 eV photon. An electron and a positron are created that each have a rest mass of 510998.896 eV (1.022 MeV combined).
The transition state orbitsphere at orbit state $n = 1/137.035999$ has a magnetic field energy equal to $2 \times 510998.896$ eV (i.e. the rest mass of the electron and the positron).

If a hydrogen atom could reach $n = 1/137.035999$ then it would have a magnetic field energy of $510998.896$ eV.

The energy required to fully ionize (i.e. overcome electrostatic force of attraction and separate out to infinity) the electron from the proton is $510998.896$ eV if the hydrogen were initially at orbit state $n = 1/137.035999$. At this orbit state, the radius is $137.035999$ times smaller than normal hydrogen in the $n = 1$ ground state.

$\text{Energy} = \text{Force} \times \text{Distance}$
The key to understanding pair production and GUTCP can be seen in Chap 29 "Pair Production" of Mills's book “The Grand Unified Theory of Classical Physics” (GUTCP) Below are two quotes from that chapter:

**Quoting from GUTCP, just below GUTCP Eq. (29.20), in chapter 29 "Pair Production“**

Thus, the LC resonance frequency of free space for a transition state electron orbitsphere equals the frequency of the photon, which forms the transition state orbitsphere. The impedance of any LC circuit goes to infinity when it is excited at the resonance frequency. **Thus, the electron transition state orbitsphere is an LC circuit excited at the corresponding resonance frequency of free space.** The impedance of free space becomes infinite, and electromagnetic radiation cannot propagate. At this event the frequency, wavelength, velocity and energy of the transition state orbitsphere equal that of the photon. The energy of the photon is equal to the rest mass energy of the particle at zero potential energy [fully ionized free electron], and charge is conserved. [emphasis added]

**And from GUTCP, just below GUTCP Eq. (29.15):**

Thus, the energy stored in the magnetic field of the transition state electron orbitsphere equals the electrical potential energy of the transition state orbitsphere. **The magnetic field is a relativistic effect of the electrical field;** thus, equivalence of the potential and magnetic energies when \( v = c \) is given by Special Relativity where these energies are calculated using Maxwell’s Equations. The energy stored in electric and magnetic fields of a photon are equivalent. The corresponding equivalent energies of the transition state orbitsphere are the electrical potential energy and the energy stored in the magnetic field of the orbitsphere. [emphasis added]
Comparison of allowed orbits in hydrogen for GUTCP and Bohr Model.

Bohr Model

Mills Model

Comparison of allowed orbits in hydrogen for GUTCP and Bohr Model.
Ionization energy for one electron atoms. Derived using GUTCP.

Ionization energy $E_B$ for one electron atoms equals

$$E_B = \frac{(\alpha Z)^2 m_e c^2}{\left(1 - \left(\frac{\alpha Z}{1 + \frac{m_e}{2m_p A}}\right)^2 + \frac{m_e}{m_p A}\right)^{\frac{1}{2}}} - m_e c^2 \left(1 - \left(\frac{\alpha Z}{1 + \frac{m_e}{2m_p A}}\right)^2\right)^{-1}$$

(1.293)

$\alpha$ = fine structure constant

$m_p$ = mass proton

$A$ = atomic mass number

$m_{e0}$ = rest mass electron

Ionization of 2 electron atoms

\[ r_2 = r_1 = a_o \left( \frac{1}{Z - 1} - \sqrt{s(s+1)} \right) ; s = \frac{1}{2} \]  

(7.35)

\[ r_n = r_n \left[ \sqrt{1 - \left( \frac{\nu}{c} \right)^2} \sin \left( \frac{\pi}{2} \left( 1 - \left( \frac{\nu}{c} \right)^2 \right)^{3/2} \right) + \frac{1}{2\pi} \cos \left( \frac{\pi}{2} \left( 1 - \left( \frac{\nu}{c} \right)^2 \right)^{3/2} \right) \right] \]  

(1.280)

\[ E(\text{electric}) = -\frac{(Z-1)e^2}{8\pi\varepsilon_0 r_1} \]  

(7.45)

\[ E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_1^3} = \frac{8\pi\mu_0 \mu_B^2}{r_1^3} \]  

(7.46)

\[ \text{Ionization Energy} = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy} \]  

(7.63)

Ionization energy for Helium

\[ r_2 = r_1 = a_0 \left( \frac{1}{Z - 1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)} \right); \quad s = \frac{1}{2} \]  

(7.35)

\[ E(\text{electric}) = - \frac{(Z-1)e^2}{8\pi\varepsilon_0 r_1} \]  

(7.45)

\[ E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_1^3} = \frac{8\pi\mu_0 \mu_B^2}{r_1^3} \]  

(7.46)

\[ \text{Ionization Energy(He)} = -E(\text{electric}) + E(\text{magnetic}) \left( 1 - \frac{1}{2} \left( \frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \]  

(7.44)
Standard Accepted Theory
Electron falls from higher orbit state to lower orbit state and emits electromagnetic radiation. Lowest possible orbit state is $n = 1$.

GUTCP
Electron falls from higher orbit state to lower orbit state and emits electromagnetic radiation and thermal kinetic energy. Lowest possible orbit state is $n = \frac{1}{137}$.

Fractional orbits are allowed, i.e. $(n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{137})$. 
Where does GUTCP diverge from the Bohr Model in terms of the derivation of the energy equations?

GUTCP is very similar to the Bohr Model in deriving the energy equations for the hydrogen atom. The Bohr Model makes the postulate that the electron has an angular momentum \( L = mvr \) equal to a \( n \hbar \) at all principal orbit states \( n \). The electric charge experienced by the electron is equal to the proton’s elementary charge \( e \) (although this is not a postulate since it is standard physics).

But …

GUTCP makes the postulate that the electric charge experienced by the electron is equal to the elementary charge \( e \) divided by the principal quantum number \( n \), or \( e/n \), and is due to the proton and the trapped photon. It also makes the postulate that the angular momentum \( L = mvr \) or mass*velocity*radius) of the electron is equal to \( \hbar \) or the reduced Planck constant at all principal orbit states \( n \).

The Bohr Model makes the postulate that the electron has an angular momentum \( L = mvr \) equal to a \( n \hbar \) at all principal orbit states \( n \). The electric charge experienced by the electron is equal to the proton’s elementary charge \( e \) (although this is not a postulate since it is standard physics).

Side Note: The postulate used in the Bohr Model can alternatively be that the circumference of the electron orbit is equal to the principal quantum number \( n \) multiplied by the electron’s de Broglie wavelength while in GUTCP it can be that the circumference of the orbit circle is equal to one electron de Broglie wavelength at all principal quantum number \( n \).
Appendix G

Capacitive Electric Energy of the TSO.
The following calculation of "Capacitive Electric Energy" for the TSO at pair production is somewhat speculative on my part and does not come directly from GUTCP. It includes a factor of 1/2 in the capacitance equation (see Eq. (G.4) in this Appendix) which might be related to the following statement in GUTCP which is for the tau and muon lepton:

Because two magnetic moments are produced, the magnetic energy (and corresponding photon frequency) in the proper frame is two times that of the electron frame. Thus, the electron time is corrected by a factor of two relative to the proper time. [emphasis added]

The quote above can be found just above GUTCP Eq. (36.7) in the chapter on Leptons. If I find out later that this "Capacitive Electric Energy" is irrelevant then I will delete this section.

The calculation of the electric energy stored in the resonant electric/magnetic oscillation of the TSO starts with the standard equation for energy stored in a capacitor

\[ E = \frac{1}{2} QV \]  \hspace{1cm} (G.1)

Where Q in Eq. (G.1) above is the charge of the electron and equals the elementary charge "-e"

\[ Q = -e \]

the equation for voltage in a capacitor is

\[ V = \frac{Q}{C} \]  \hspace{1cm} (G.2)
But “Q” in Eq. (G.2) is the electric field that the electron experiences. For the hydrogen atom, this field is not simply the charge of the proton $e$ but is instead equal to $e/n$ because in GUTCP the “trapped photon” alters the electric field experienced by the electron by a factor of $1/n$. Since this is a derivation for the TSO which does not have a trapped photon, GUTCP inserts a factor of $1/n$ into the energy equation and writes it "arises from Gauss' law surface integral and the relativistic invariance of charge" (see text right above GUTCP Eq. (29.10) in GUTCP).

With this $1/n$ factor, Eq. (G.2) becomes

$$V = \frac{Q}{C} = \frac{(e/n)}{C}$$  \hspace{1cm} (G.3)

The capacitance of an isolated sphere of radius $r$ is

$$C = 4\pi\varepsilon_0 r$$

inserting the radius of the TSO

$$r = na_0 = \alpha a_0 \text{ (radius of TSO)}$$

gives

$$C = 4\pi\varepsilon_0 \alpha a_0$$

At this point, the only way to make the final energy equation work is to include a factor of $1/2$.

I am speculating that the $\frac{1}{2}$ factor comes from the fact that $\frac{1}{2}$ the volume of the TSO resonates due to the electron and $\frac{1}{2}$ the volume resonates due to the positron (the anti-electron).
The capacitance then becomes

\[ C = \left(\frac{1}{2}\right) 4\pi \varepsilon_0 \alpha a_0 = 2\pi \varepsilon_0 \alpha a_0 \]  

(G.4)

Putting Eqs. (G.4) and (G.3) into Eq. (G.1) with principal quantum number \( n = \alpha \) gives

\[
E = \frac{1}{2} QV = \frac{1}{2} e \left( \frac{(e/n)}{2\pi \alpha a_0 \varepsilon_0} \right) = \frac{e^2}{4\pi \alpha^2 a_0 \varepsilon_0} = m_0 c^2 = 510998.896 \text{ eV}
\]

Capacitive Electric Energy

Rest mass of the electron.
Appendix H

Capacitive Electric Energy of the TSO For a positron/electron pair (speculative, not in GUTCP).

*Portions of the following note are also on page 16 of this document: The central electric field between the positron and the electron at particle production most likely can not be modeled using conventional equations for a static, steady state electric field. The reason is particle production is not a steady state process and the positron and electron form from a photon and immediately ionize to a free positron and a free electron. So describing the positron as supplying the central electric field that replaces the proton in the hydrogen atom is most likely incorrect and oversimplifying the math. The calculation of the electric field in this appendix could be used as a tool for determining the correct equations for the electric field between the positron and electron but I don’t think the equations are correct as they are now written in this appendix.
Hypothesis: Assume the positron and electron take the form of a spherical capacitor where the positron and electron have different radii and the electric field is in between the two spheres. The formula for the capacitance of a spherical capacitor is

\[ C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} \]  
\[ \text{(H.1)} \]

The calculation of the electric energy stored in a capacitor is

\[ E = \frac{1}{2}QV \]  
\[ \text{(H.2)} \]  
(This applies to a capacitor of any shape)
Where

\[ Q \] is the sum of the charge of the electron and the positron which equals \( e + e = 2e \) which gives

\[ Q = 2e \]

and \( V \) is the voltage

[Note: The positron + electron charge does not sum to zero! In other words, it is not: \( e + (-e) = 0 \)]

Eq. (H.1) becomes

\[ E = \frac{1}{2} (2e)V \]  \hspace{1cm} (H.3)

the equation for voltage in a capacitor is

\[ V = \frac{Q}{C} \]  \hspace{1cm} (H.4)

But \( Q \) in Eq. (H.4) is the electric field that the electron experiences. For the hydrogen atom, this field is not simply the charge of the proton \( e \) but is instead equal to \( e/n \) because in GUTCP the "trapped photon" alters the electric field experienced by the electron by a factor of \( 1/n \). Since this is a derivation for the positron/electron production (or the TSO, the transition state orbitsphere) which does not have a trapped photon, GUTCP inserts a factor of \( 1/n \) into the energy equation and writes it "arises from Gauss' law surface integral and the relativistic invariance of charge" (see text right above GUTCP Eq. (29.10) in GUTCP).
This \( \frac{1}{n} \) factor can be applied to the voltage equation so Eq. (H.4) becomes

\[
V = \frac{Q}{C} = \frac{(2e/n)}{C} \tag{H.5}
\]

If the electron at particle production is assumed to be \textit{larger} than the positron then it would be radius \( b \) in Figure (H.1) and assuming the electron has the radius of the TSO orbitsphere:

\[
b = r = na_0 = \alpha a_0 = 3.861592643 \times 10^{-13} \text{ m} \tag{H.6}
\]

Where

\[
a_0 = \text{the bohr radius} = 5.291772083 \times 10^{-11} \text{ m}
\]

\[
\alpha = \text{the fine structure constant} = 1/137.035999
\]

The positron at particle production is then assumed to be \textit{smaller} than the electron and would be radius \( a \) in Figure (H1).

Inserting these into Eq. (H.1) for capacitance gives

\[
C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{\alpha a_0}\right)} \tag{H.7}
\]

Where \( a \) in the equation above is the radius for the positron and is the unknown radius which we are trying to calculate.
Inserting Eq. (H.7) into Eq. (H.5) and setting $n = \alpha = \alpha$ gives an equation for voltage $V$

$$V = \frac{Q}{C} = \frac{(2e/n)}{\left(\frac{4\pi\varepsilon_0}{1 - \frac{1}{a \alpha a_0}}\right)} = \frac{(2e/\alpha)}{\left(\frac{4\pi\varepsilon_0}{1 - \frac{1}{a \alpha a_0}}\right)}$$  \hspace{1cm} (H.8)

Inserting voltage Eq. (H.8) into energy Eq. (H.2) gives the equation for energy

$$E = \frac{1}{2} (2e)V = \frac{1}{2} (2e)^* \frac{(2e/\alpha)}{\left(\frac{4\pi\varepsilon_0}{1 - \frac{1}{a \alpha a_0}}\right)} = \frac{(2e^2/\alpha)}{\left(\frac{4\pi\varepsilon_0}{1 - \frac{1}{a \alpha a_0}}\right)}$$

Which when rearranged becomes

$$\frac{1}{a} - \frac{1}{\alpha a_0} = \frac{4\pi\varepsilon_0 E}{(2e^2/\alpha)}$$  \hspace{1cm} (H.9)
And solving for positron radius \( a \) results in

\[
a = \left( \frac{1}{\alpha a_0} + \frac{4\pi \varepsilon_0 E}{(2e^2/\alpha)} \right)^{-1} \quad \text{(H.10)}
\]

The energy \( E \) in the equation above is equal to the electron and the positron mass energy which equals \( 2 \times 510998.896 \text{ eV} = 1021997.792 \text{ eV} \) or converting to Joules is \( 1.637420876 \times 10^{-13} \text{ J} \).

In Eq. (H.10):
- \( e = \text{elementary charge} = 1.602176487 \times 10^{-19} \text{ Coulomb} \)
- \( \varepsilon_0 = \text{permittivity of free space} = 8.854187817 \times 10^{-12} \text{ Coulomb/Nm2} \)
- \( a_0 = \text{bohr radius} = 5.291772083 \times 10^{-11} \text{ m} \)
- \( E = \text{mass energy of positron + electron} = 1.637420876 \times 10^{-13} \text{ J} \)
- \( \alpha = \text{alpha} = 0.007297352538 \) (unitless) \( \approx \frac{1}{137.035999} \)
Inserting the constants into Eq. (H.10) and calculating the positron radius \( a \) gives
\[
a = 1.930796322 \times 10^{-13} \text{ m}
\]
The electron radius from Eq. (H.6) is
\[
b = 3.861592643 \times 10^{-13} \text{ m}
\]
The ratio of the electron radius \( b \) to the positron radius \( a \) is
\[
b/a = 2.0000000
\]
In this hypothesis, the radius of the electron is twice the radius of the positron at particle production.

Changing the above equations by assuming the positron is larger than the electron (with the electron at \( r = \alpha a_0 = 3.861592643 \times 10^{-13} \text{ m} \)) doesn't seem to work because it results in the positron radius having effectively zero radius. This seems to be due to the fact that the fine structure constant \( \alpha \) (alpha) was inserted at crucial locations in the equations.

Examining equation (H.10) shows that \( \alpha \) was inserted into the denominator of one term and the numerator of another term and is the key to the final result:

\[
a = \left( \frac{1}{\alpha a_0} + \frac{4\pi \varepsilon_0 E}{(2e^2/\alpha)} \right)^{-1} = \left( \frac{1}{\alpha a_0} + \frac{\alpha 4\pi \varepsilon_0 E}{2e^2} \right)^{-1} \quad \text{ (H.10)}
\]
Appendix J

(appendix I does not exist in this document so that it does not create confusion with equations from the Introduction in GUTCP)

Young’s Double Slit Experiment.

Disclaimer: This appendix shows mathematical results of Young’s double slit experiment and Bragg’s Law that may not be novel and most likely can be found in the literature. I did a brief internet search and some equations and discussions written by others may match the content in this appendix.
Summary:
This appendix will show that the pattern of bright and dark fringes in Young’s double slit experiment can be obtained if a hypothetical photon (designated photon-B) deflects the photons (designated photon-A) passing through the slit. Photon-B travels perpendicular to photon-A and has a wavelength and mass equal to:

\[ \lambda_B = \frac{d}{n} \quad \text{Eq. (J.12)} \]

\[ m_B = \frac{nh}{dc} \quad \text{Eq. (J.10)} \]

where

- \( \lambda_B \) = wavelength of photon-B which is absorbed by photon-A or emitted by photon-A
- \( d \) = separation distance between slits
- \( n \) = positive integer, 1,2,3 etc.
- \( h \) = Planck’s constant
- \( c \) = speed of light

Note: The equation for mass, \( m_B \), in Eq. (J.10) is based on the de Broglie wavelength equation applied to the wavelength equation from Eq. (J.12).

In this hypothetical scenario, the wavelength and mass of photon-B are not a function of any parameter connected with photon-A, yet the mathematical results match the Young’s double slit experiment. Photon-B parameters are only a function of the slit separation distance \( d \) and standard physics constants such as speed of light \( c \) and Planck’s constant \( h \) and an integer \( n \).
Summary continued:

The commonly accepted theory for the results of the double slit experiment is that the bright and dark fringes are due to the photon interfering with itself due to constructive and destructive interference of its wavelength. But Mills’s GUTCP makes the (correct) point that this violates conservation of energy because it implies that photons (energy) can be destroyed. This appendix does not try to describe the GUTCP explanation for the photon double slit experiment since I am still trying to understand it fully.

Towards the end of this appendix it is further hypothesized that Surface Plasmon Polaritons (SPP's) could cause the deflection of photon-A and create the bright and dark fringes in the photon double slit experiment. This SPP explanation is not directly related to photon-B described above but it is indirectly related since the math (specifically the momentum equations) does match. Also, the SPP’s could explain Bragg’s Law and Low Energy Electron Diffraction (LEED) also known as the Davison Germer Experiment.
Young’s double slit experiment

Image below from:
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/slits.html

Figure J1.

The notation on the next page matches the notation above except fringe number $m$ is replaced with $n$ so that it is not confused with mass.
In Young's double slit experiment, light passing through two slits creates a pattern of bright and dark fringes on a distant screen. The standard accepted theory is that this is due to constructive and destructive interference of the light waves due to different path lengths from each slit to some point on the opposing screen. The results of the double slit experiment gave an equation for the location of each bright fringe on the opposing screen:

\[ y \approx \frac{n\lambda_A D}{d} \]  \hspace{1cm} (J.1)

where

- \( y \) = distance to each bright fringe from the middle of the screen
- \( \lambda_A \) = wavelength of laser light source (coming from left in Figure (J2) and designated photon-A)
- \( D \) = distance between slits and screen
- \( d \) = separation distance between two slits
- \( n \) = bright fringe number

This appendix will attempt to show that the fringes could be created in a different way. Specifically, Eq. (J.1) could be obtained if the photon passing through the slit emits or absorbs a second photon that travels perpendicular to the original photon. This is a hypothetical scenario since photons are not known spontaneously absorb or emit secondary photons without first interacting with matter. In this hypothetical scenario described on the following pages, the original photon traveling through the slit is designated photon-A and the emitted or absorbed photon that travels perpendicular to it is designated photon-B.

Towards the end of this appendix it is hypothesized that Surface Plasmon Polaritons (SPP's) could cause the deflection of the photon and create the bright and dark fringes in the photon double slit experiment. Also, the SPP’s could explain Bragg’s Law and Low Energy Electron Diffraction (LEED) also known as the Davison Germer Experiment.
Young’s double slit experiment

Photons (designated photon-A) created by laser.

- Mass: \( m_A \)
- Wavelength: \( \lambda_A \)
- Linear momentum: \( p = m_A c \)

\[ d = \text{slit separation distance} \]

\[ D = \text{centerline distance between slits and screen} \]

\[ n = \text{fringe number} \]

\[ \tan \theta \approx \sin \theta \approx \frac{y}{D} \]

\[ y = \text{distance to each bright fringe.} \]

Experimental result:

\[ y \approx \frac{n \lambda_A D}{d} \]

Bright fringe lines represented by this shape:

Figure J2.
The photon coming from the laser source is designated photon-A. It is deflected at the slits and creates an angle with the horizontal designated $\theta$. The linear momentum of a photon is equal to its mass $m$ multiplied by the speed of light $c$: $p = mc$. The mass of the photon can be calculated from its energy using Einstein energy equation $E = mc^2$. Therefore, if the mass of photon-A is designated $m_A$ then its linear momentum is equal to $p = m_A c$. If it is assumed that the linear momentum of photon-A is unchanged along the X-axis direction after passing through the slit but it gains or loses momentum in the Y-axis direction then the projected momentum vectors for the deflected photon-A for the X and Y axes directions are as shown in Figure (J3) below as bright green arrows.

Photons (designated photon-A) created by laser.

- **mass** = $m_A$
- **wavelength** = $\lambda_A$
- **linear momentum**: $p = m_A c$

$D =$ slit separation distance

$n =$ fringe number

$p_x = m_A c$

$p_y = m_A c \tan \theta$

$y =$ distance to each bright fringe.

Figure J3.
Since the angle $\theta$ is small, the tangent of $\theta$ is approximately equal to sine of $\theta$:

$$\tan \theta \approx \sin \theta \approx \frac{y}{D}$$

Note: without this approximation, the following slightly more complicated equation would have to be used as the sine of the angle:

$$\sin \theta = \frac{y}{\sqrt{y^2 + D^2}}$$

Diagram from previous page redrawn using the approximation:

$$\tan \theta \approx \sin \theta \approx \frac{y}{D}$$

Figure J4.

- **Photons (designated photon-A) created by laser.**
  - Mass = $m_A$
  - Wavelength = $\lambda_A$
  - Linear momentum: $p = m_A c$
  - $p_y = m_A c \sin \theta$
  - $y = \text{distance to each bright fringe.}$
  - $D = \text{distance between slits and screen}$
  - $n=0, 1, 2$
The derivation on the next 3 pages are based on Figures (J3) and (J4) and derives the equation for the Y-axis momentum vector of photon-A in terms of the slit separation distance d.

The photons coming from the left in Figure (J2) are initially traveling in the X-axis direction and get deflected at the slits so that it creates an angle with the X-axis. The experimental results show a pattern of bright and dark fringes that have a y coordinate according to the equation:

\[ y \approx \frac{n\lambda_A D}{d} \]  \hspace{1cm} (J.1)

where:

- \( y \) = distance to each bright fringe from the middle of the screen
- \( \lambda_A \) = wavelength of laser light source (coming from left in Figure (J2) and designated photon-A)
- \( D \) = distance between slits and screen
- \( d \) = separation distance between slits
- \( n \) = bright fringe number

Screen distance \( D \) can be eliminated by putting Eq. (J.1) in terms of the trigonometric function sine using the approximation that the sine of a small angle is approximately equal to the tangent of the angle:

\[ \tan \theta \approx \sin \theta \approx \frac{y}{D} \]  \hspace{1cm} (J.2)

which can be written as

\[ y \approx D \sin \theta \]  \hspace{1cm} (J.3)

inserting Eq. (J.3) into Eq. (J.1) gives

\[ D \sin \theta = \frac{n\lambda_A D}{d} \]  \hspace{1cm} (J.4)
which reduces to

\[ \sin \theta = \frac{n \lambda_A}{d} \]  \hspace{1cm} (J.5)

The de Broglie wavelength equation for photon-A with mass \( m_A \) is:

\[ \lambda = \frac{h}{p} = \frac{h}{m_A c} \]  \hspace{1cm} (J.6)

inserting Eq. (J.6) into Eq. (J.5) gives

\[ \sin \theta = \frac{nh}{dm_A c} \]  \hspace{1cm} (J.7)

moving the momentum \( m_A c \) to the left side gives

\[ m_A c \sin \theta = \frac{nh}{d} \]  \hspace{1cm} (J.8)

It can be seen in Figure (J4) that Eq. (J.8) is the Y-axis momentum of photon-A after it passes through a slit. This Y-axis momentum vector is equal to an integer \( n \) multiplied by Planck’s constant \( h \) divided by the slit separation distance \( d \). Hypothetically it is possible that this vertical momentum is created by a second photon with unknown origin traveling perpendicular to photon-A which is absorbed by photon-A or emitted by photon-A. In this scenario, the second photon is designated photon-B and must have a linear momentum according to Eq. (J.8)

\[ p = m_B c = \frac{nh}{d} \]  \hspace{1cm} (J.9)

where \( m_B \) is the mass of photon-B
Solving Eq. (J.9) for mass gives

\[ m_B = \frac{nh}{dc} \]  

(J.10)

The de Broglie relation is:

\[ \lambda = \frac{h}{p} = \frac{h}{mc} \]  

(J.11)

Inserting Eq. (J.10) into Eq. (J.11) gives the wavelength of photon-B:

\[ \lambda_B = \frac{h}{m_B c} = \frac{dc h}{nh c} = \frac{d}{n} \]  

(J.12)

Wavelength is related to photon radius by

\[ \lambda = 2\pi r \]

and

\[ r = \frac{\lambda}{2\pi} \]

Which gives the radius of photon-B:

\[ r_B = \frac{\lambda}{2\pi} = \frac{d}{n2\pi} \]  

(J.13)

And the diameter:

\[ \text{Diameter} = 2r = \frac{d}{n\pi} \]  

(J.14)

A graphical representation of this photon on the slits in the double slit experiment using properly scaled sizes is shown in Figure (J5).
Young’s Double Slit experiment is based on the deflection of the initial photon (designated photon-A) which travels through one of two slits. If traveling through the slits somehow creates photon-B which travels perpendicular to photon-A and deflects it in the opposite direction then photon-B is not a function of any parameter related to photon-A. The mass of photon-B is only a function of the slit separation distance $d$, Planck’s constant $h$, the speed of light $c$ and a positive integer $n$.

<table>
<thead>
<tr>
<th>photon-B, for all fringes:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelength: $\lambda_B = \frac{d}{n}$</td>
<td>Eq. (J.12)</td>
</tr>
<tr>
<td>radius: $r_B = \frac{d}{n2\pi}$</td>
<td>Eq. (J.13)</td>
</tr>
<tr>
<td>mass: $m_B = \frac{nh}{dc}$</td>
<td>Eq. (J.10)</td>
</tr>
</tbody>
</table>

Figure J5.
Young’s Double Slit experiment is based on the deflection of the initial photon (designated photon-A) which travels through one of two slits. If traveling through the slits somehow creates photon-B which travels perpendicular to photon-A and deflects it in the opposite direction then photon-B is not a function of any parameter related to photon-A. The mass of photon-B is only a function of the slit separation distance $d$, Planck’s constant $h$, the speed of light $c$ and a positive integer $n$.

Photon-B, for fringe $n = 2$

- wavelength = $\lambda_B = \frac{d}{2}$
- radius = $r_B = \frac{d}{4\pi}$
- Diameter = $\frac{d}{2\pi}$

 photon-B, for all fringes:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelength</td>
<td>$\lambda_B = \frac{d}{n}$</td>
<td>(J.12)</td>
</tr>
<tr>
<td>radius</td>
<td>$r_B = \frac{d}{n2\pi}$</td>
<td>(J.13)</td>
</tr>
<tr>
<td>mass</td>
<td>$m_B = \frac{nh}{dc}$</td>
<td>(J.10)</td>
</tr>
</tbody>
</table>

Diameter of circle matches diameter of photon-B for $n = 2$ and is properly scaled relative to slit separation distance $d$. The circumference of the circle is equal to the wavelength and equal ½ of the slit separation distance $d$.

Figure J6.
Speculation: The double slit experiment results could be from the following scenario:

Imagine a line of mass and charge (similar to “infinitesimal masses and charges” in GUTCP orbitsphere) having a length \( d \) equal to the slit separation distance and stretched out between the two slits. Now imagine them “rolling up” and forming a photon having a circumference equal to \( d \). Next imagine them colliding inelastically (or maybe elastically) with photon-A in a perpendicular direction as photon-A crossed through the slits.

<table>
<thead>
<tr>
<th>photon-B, for all fringes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelength = ( \lambda_B = \frac{d}{n} ) \hspace{1cm} Eq. (J.12)</td>
</tr>
<tr>
<td>radius = ( r_B = \frac{d}{n2\pi} ) \hspace{1cm} Eq. (J.13)</td>
</tr>
<tr>
<td>mass = ( m_B = \frac{nh}{dc} ) \hspace{1cm} Eq. (J.10)</td>
</tr>
</tbody>
</table>

Figure J7.
If the photon sizes for $n = 1$ and $n = 2$ are laid out side by side between the slits with proper scaling.

**Photon-B, for fringe $n = 1$**
- Wavelength: $\lambda_B = d$
- Diameter: $\frac{d}{\pi}$

**Photon-B, for fringe $n = 2$**
- Wavelength: $\lambda_B = \frac{d}{2}$
- Diameter: $\frac{d}{2\pi}$

$d =$ slit separation distance

Figure J8.
### Diffraction with photons, electrons and x-rays give (almost) the same equation.

<table>
<thead>
<tr>
<th>Photon double slit experiment</th>
<th>Photon double slit experiment</th>
<th>equation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Young's double slit experiment:</em> Photons of light deflected and create measured photon peaks on opposing screen.</td>
<td>[ n\lambda = d \sin \theta ]</td>
<td>(same as Eq. (J.5))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electron diffraction</th>
<th>Electron diffraction</th>
<th>equation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Davison Germer Experiment:</em> Low energy electron diffraction off a nickel surface create measured reflection peaks on opposing screen. Also known as Low Energy Electron Diffraction (LEED).</td>
<td>[ n\lambda = d \sin \theta ]</td>
<td>(same as Eq. (J.5))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X-ray diffraction</th>
<th>X-ray diffraction</th>
<th>equation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Bragg’s Condition:</em> X-ray diffraction off crystal planes create measured reflection peaks on opposing screen.</td>
<td>[ n\lambda = 2d \sin \theta ]</td>
<td>(same as Eq. (J.5) except for “2”)</td>
</tr>
</tbody>
</table>
As shown on the previous page, there is a “2” in the x-ray diffraction equation which is not in the photon double slit equation or the electron surface diffraction equation:

\[
\text{X-ray Diffraction:} \quad n\lambda = 2d \sin \theta
\]

But if the distance \( d \) is taken to be the distance between two crystal planes (see figure at right), then the x-ray diffraction equation matches the double slit experiment equation and the electron surface diffraction equation:

\[
\text{X-ray Diffraction:} \quad n\lambda = d \sin \theta
\]
**Surface plasmon polaritons (SPP’s)**

Assuming photons are spheres with circumferences that match their wavelengths then the diameter of photon-B is a factor of 3.14 (i.e. $\pi$) smaller than the distance between the slits.

**If photon-B exists then how does it get created?**

Photon-B may *somehow* be related to *surface plasmon polaritons* (SPP’s) which are electromagnetic waves that travel along a surface:

The following is from Wikipedia and I highlighted some text in red:

Surface plasmon polaritons (SPPs), are infrared or visible-frequency electromagnetic waves, which travel along a metal-dielectric or metal-air interface. The term "surface plasmon polariton" explains that the wave involves both charge motion in the metal ("surface plasmon") and electromagnetic waves in the air or dielectric ("polariton").

They are a type of surface wave, guided along the interface in much the same way that light can be guided by an optical fiber. [...] Perpendicular to the interface, they have subwavelength-scale confinement. An SPP will propagate along the interface until its energy is lost either to absorption in the metal or scattering into other directions (such as into free space).

 [...] 

SPPs can be excited by both electrons and photons. Excitation by electrons is created by firing electrons into the bulk of a metal. As the electrons scatter, energy is transferred into the bulk plasma. The component of the scattering vector parallel to the surface results in the formation of a surface plasmon polariton.

 [...] 

For a photon to excite an SPP, both must have the same frequency and momentum.

So, photon-B as described could be a surface plasmon polariton (SPP). And the results of the photon double slit experiment, electron surface diffraction experiment and the x-ray diffraction experiment could all be due to surface plasmon polaritons.
There are different types of polaritons:

**Polaritons:**
Coupled state between an elementary excitation and a photon.

**Plasmon polariton:** coupled state between a plasmon and a photon.

**Phonon polariton:** coupled state between a phonon and a photon.

In physics, polaritons are quasiparticles resulting from strong coupling of electromagnetic waves with an electric or magnetic dipole-carrying excitation. [...] A polariton is the result of the mixing of a photon with an excitation of a material. The following are types of polaritons:

- **Phonon polaritons** result from coupling of an infrared photon with an optic phonon.
- **Exciton polaritons** result from coupling of visible light with an exciton.
- **Intersubband polaritons** result from coupling of an infrared or terahertz photon with an intersubband excitation.
- **Surface plasmon polaritons** result from coupling of surface plasmons with light (the wavelength depends on the substance and its geometry).
- **Bragg polaritons** ("Braggoritons") result from coupling of Bragg photon modes with bulk excitons.
Could photon-B be a resonant condition between the two slits?

**Surface plasmon resonance** (SPR) is the resonant oscillation of conduction electrons at the interface between a negative and positive permittivity material stimulated by incident light. The resonance condition is established when the frequency of incident photons matches the natural frequency of surface electrons oscillating against the restoring force of positive nuclei. SPR in subwavelength scale nanostructures can be polaritonic or plasmonic in nature. [...] The surface plasmon polariton is a non-radiative electromagnetic surface wave that propagates in a direction parallel to the negative permittivity/dielectric material interface. Since the wave is on the boundary of the conductor and the external medium (air, water or vacuum for example), these oscillations are very sensitive to any change of this boundary, such as the adsorption of molecules to the conducting surface.
Conclusions:
This appendix shows that the pattern of bright and dark fringes in Young’s double slit experiment could be due to one of the following causes:

1. A photon (designated photon-B in this Appendix) is emitted or absorbed by the original photon passing through the slit causing it to deflect and produce the bright and dark fringes. Photon-B has a linear momentum that is only a function of the slit separation distance “d” and standard physics constants (see Eq. (J.9)). Also, photon-B could explain Bragg’s Law and Low Energy Electron Diffraction (LEED) also known as the Davison Germer Experiment.

2. **Surface plasmon polaritons** (SPP’s) which are electromagnetic waves that travel along a surface could be created which cause the original photon to deflect and produce the bright and dark fringes. Also, SPP’s could explain Bragg’s Law and Low Energy Electron Diffraction (LEED) also known as the Davison Germer Experiment.