Measures, Shape and Space

Introduction to Trigonometric Ratios

A. Basic knowledge

ratio

(a) $3:4$ can be written as $\frac{3}{4}$

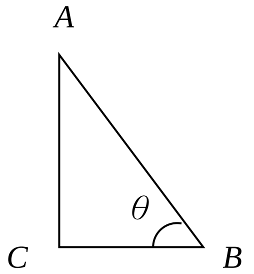
(b) $\frac{5}{8}$ can be written as $5:8$

equal ratios

(a) $2:3=4:6$

(b) $10:25=2:5$

Pythagoras' theorem



$$AC^2+BC^2=AB^2$$

B. Trigonometric ratios

- The term "trigonometry" is originated from Greek. It means measuring triangles.
- In the above diagram, the side which is opposite to the right angle (i.e. *AB*) is called the hypotenuse. It is the longest side in ΔABC.
- The side which is opposite to the angle θ (i.e. *AC*) is called the opposite side.
- The shorter side which is adjacent to the angle θ (i.e. *BC*) is called the adjacent side.
- The ratio $\frac{adjacent}{hypotenuse}$ is called the cosine ratio for angle θ.

$$\cos\theta=\frac{adjacent}{hypotenuse}$$

- The ratio $\frac{opposite}{hypotenuse}$ is called the sine ratio for angle θ.

$$\sin\theta=\frac{opposite}{hypotenuse}$$

- The ratio $\frac{opposite}{adjacent}$ is called the tangent ratio for angle θ.

$$\tan\theta = \frac{opposite}{adjacent}$$

- We call $\cos\theta$, $\sin\theta$ and $\tan\theta$ the trigonometric ratios.

C. The cosine ratio for an angle

- In the above figure, the cosine ratio for the angle θ is $\frac{adjacent}{hypotenuse}$, thus

  $\cos\theta = \frac{BC}{AB}$.

- If θ varies between $0°$ and $90°$, the value of $\cos\theta$ varies between 0 and 1.
- With a calculator, we can find $\cos\theta$ if θ is known (and vice versa).
- We will use the cosine ratio for an angle to solve some problems involving right-angled triangles (e.g. finding angles and lengths).

D. The sine ratio for an angle

- In the above figure, the sine ratio for the angle θ is $\frac{opposite}{hypotenuse}$, thus

  $\sin\theta = \frac{AC}{AB}$.

- If θ varies between $0°$ and $90°$, the value of $\sin\theta$ varies between 0 and 1.
- With a calculator, we can find $\sin\theta$ if θ is known (and vice versa).
- We will use the sine ratio for an angle to solve some problems involving right-angled triangles (e.g. finding angles and lengths).

E. The tangent ratio for an angle

- In the above figure, the tangent ratio for the angle θ is $\frac{opposite}{adjacent}$, thus

  $\tan\theta = \frac{AC}{BC}$.

- Similar to the cosine and the sine ratios, we can find, using a calculator, $\tan\theta$ if θ is known (and vice versa).
- We will use the sine ratio for an angle to solve some problems involving right-angled triangles (e.g. finding angles and lengths).

F. Exercises

1. Find the values of the following.

(a) $\cos 30° + \cos 40°$

(b) $\cos(30° + 40°)$

(c) $2\cos 35°$

(d) $\cos(2 \times 35°)$

2. Find the values of the following.

(a) $\sin 50° - \sin 10°$

(b) $\sin(50° - 10°)$

(c) $3\sin 18°$

(d) $\sin(3 \times 18°)$

3. Find the value of $\frac{1}{2}\cos 10°$. If $\cos\theta = \frac{1}{2}\cos 10°$, find θ.

4. Find the value of $\frac{1}{3}\sin 15°$. If $\sin\theta = \frac{1}{3}\sin 15°$, find θ.

5. Find θ in the following.

(a) $\cos\theta = \cos 50° + \cos 70°$

(b) $\cos\theta = \frac{1}{2}\cos 45° + \frac{1}{3}\cos 55°$

(c) $\cos(\theta + 20°) = 0.62$

(d) $\cos(2\theta + 30°) = 0.2345$

6. Find θ in the following.

(a) $\sin\theta = \sin 18° + \sin 38°$

(b) $\sin\theta = \frac{1}{2}\sin 56° - \frac{1}{3}\sin 14°$

(c) $\sin(\theta - 30°) = 0.772$

(d) $\sin(2\theta + 28°) = 0.9816$

7. In each of the following, find the values of A, B, and $A + B$.

(a) $\tan A = \frac{1}{2}$, $\tan B = 2$

(b) $\tan A = \frac{3}{4}$, $\tan B = \frac{4}{3}$

(c) $\tan A = \frac{2}{3}$, $\tan B = \frac{3}{2}$

8. A coin is rolling down from 12 cm long slant board to the horizontal ground. If the angle between the slant board and the horizontal is $40°$, what is the horizontal distance travelled by the coin?

9. A ladder leans against a vertical wall. The angle between the ladder and the horizontal ground is $68°$. If the distance between the foot of the ladder and the bottom of the wall is 0.8 m, find the length of the ladder.

10. The diagonal of a rectangular board in a classroom is 3.5 m. It makes an angle of $38°$ with the bottom edge of the board. If the bottom edge of the board is 1 m above the floor, what is the distance of the top of the board above the floor?

11. A child is flying a kite. The string is 300 m long and makes an angle of $47°$ with the horizontal. If the hands of the child is 0.8 m above the groud, find the height of the kite above the ground.

12. A post is opposite to a tree on the other side of a river. At a position 100 m from the post along the bank, the angle between the tree and the post is $25°$. Find the width of the river.

13. A wire is used to support a telegraph post. One end of it is 8 m above the ground and the other end is 3 m from the post. Find the angle that the wire makes with the ground.